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Supersymmetric D-branes in type-IIB plane wave background

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Abstract: We systematically analyze supersymmetric D-branes in the type-IIB plane wave background using Green-Schwarz superstring theory. We find several new supersymmetric oblique and curved D-branes. The supersymmetries preserved by various configurations of these D-branes including their intersections are also identified. In particular, we show that D_+-branes of type $(+,-,n,n)$ for $n = 1,2,3,4$ preserve 4 dynamical supersymmetries by introducing gauge field excitations and newly discovered oblique $D5$- and $D7$-branes also preserve four or two dynamical supersymmetries with gauge field condensates. Furthermore we find new curved D-branes preserving four dynamical supersymmetries.

Keywords: D-branes, Penrose limit and pp-wave background.

JHEP03(2004)058

Contents

1. Introduction

In many recent developments in string theory, D-branes carrying Ramond-Ramond charges have played an important role in the understanding of string dualities [1, 2]. D-branes have precisely the correct properties to fill out duality multiplets together with fundamental string states and other field theoretic solitons. Furthermore D-branes have been successfully used to explain various supersymmetric and nonsupersymmetric field theories including the AdS/CFT correspondence and the entropy of some black-holes. More recently, there have been attempts in understanding some cosmological issues and the hierarchy problem with D-branes.

D-branes can be described by boundary states of closed string states [3, 4]. The symmetries that the boundary state preserves are thus generically the combinations of the closed string symmetries that leave the boundary state invariant. However, it is a challenging problem to completely classify the D-branes in a general string background since it is necessary to quantize the string theory in the background. Recently a maximally supersymmetric type-IIB string background was found, which is the Penrose limit of the $AdS_5 \times S^5$ background in type-IIB supergravity [5],

$$\begin{aligned} ds^2 &= -2dx^+dx^- - \mu^2 x_I^2 (dx^+)^2 + dx_I{}^2 \,, \\ F_{+1234} &= F_{+5678} = 2\mu \,. \end{aligned} \tag{1.1}$$

Since the string theory is exactly solvable in the Ramond-Ramond background (1.1) [6, 7], it may be possible to get the complete spectrum of D-branes in the background. Moreover it was realized in [8] that the type-IIB string theory in the plane wave background (1.1) has a very simple description in terms of the dual supersymmetric Yang-Mills theory.

As the first step towards this goal, we systematically classify *static* D-branes in the maximally supersymmetric type-IIB plane wave background (1.1) using the Green-Schwarz superstring theory. Recently it was noticed in [9, 10] that the background (1.1) admits oblique D-branes (OD-branes) whose isometry is a subgroup of the diagonal SO(4) symmetry of the background as well as curved D-branes preserving some supersymmetries. The supergravity solution of oblique D-branes was also discussed in [11]. These branes do not belong to the class of the $D_\pm$-brane [12]–[20]. Only a special class of oblique and curved D-branes, however, was found in [9, 10] and it is thus demanded to know the complete list of the D-branes in the background (1.1) and the supersymmetries preserved by various configurations of these D-branes.

In this paper we are using the light-cone open string theory, where the light-cone worldvolume coordinates $X^\pm$ necessarily satisfy the Neumann boundary condition - longitudinal branes. The instantonic branes and branes with only one light-cone coordinate along the worldvolume in [14] are not visible in the light-cone open string theory. Thus these D-branes lie outside our classification. To study these branes, one may use boundary state formalism for the former branes and covariant gauge formulation for the latter.

This paper is organized as follows. In section 2, we present a worldsheet formulation using the Green-Schwarz superstring action in light-cone gauge for an open string attached on a flat D-brane. We first find the most general condition satisfied by the longitudinal D-brane in the plane wave background (1.1) and determine the complete spectrum of the flat D-branes. Supersymmetric flat longitudinal D-branes are summarized in table 1.

In section 3, we give the mode expansion of open strings consistent with a general class of open string boundary conditions in the Green-Schwarz superstring theory context.

In section 4, the analysis is generalized to the case of intersecting D-branes using the formalism of our previous work [21]. It is shown that oblique D-branes consistently intersect with usual $D_\pm$-branes.

In section 5, the supersymmetries preserved by various flat D-branes are explicitly identified by finding conserved worldsheet supercurrents consistent with open string boundary conditions [20]. In particular, we show that D_+-branes of type $(+,-,n,n)$ for $n=1,2,3,4$ preserve 4 dynamical supersymmetries by introducing gauge field excitations and newly discovered oblique $D5$- and $D7$-branes also preserve four or two dynamical supersymmetries with gauge field condensates. Furthermore, we show that D-branes with odd number of oblique directions preserve no supersymmetry and the D-branes with even number of oblique directions preserve 8 kinematical supersymmetries of which 4 supersymmetries are descending from the closed string and another 4 supersymmetries are the new kind of supersymmetry, not descending from the closed string, as identified by Skenderis and Taylor [17] for D_+-branes. Unbroken supersymmetry of these D-branes is summarized in table 2.

In section 6, the supersymmetry analysis is generalized to intersecting D-branes. All supersymmetric intersecting branes are classified in table 3.

JHEP03(2004)058

In section 7, we also discuss supersymmetric curved D-branes including those not mentioned in the previous literatures. We find newly discovered curved D-branes preserve four dynamical supersymmetries. Curved D-branes preserving dynamical supersymmetries are listed in table 4.

In section 8, we briefly review our results obtained and discuss some related issues.

In appendix A, we show that Born-Infeld fluxes introduced on the worldsheet to enhance the dynamical supersymmetry are consistent with the equation of motion for a worldvolume gauge field on a D5-brane.

2. Flat D-branes in a plane wave background

The Green-Schwarz light-cone action in the plane wave background (1.1) describes eight free massive bosons and fermions [6]. In the light-cone gauge, $X^+ = \tau$, the action is given by

$$S = \frac{1}{2\pi\alpha' p^+}\int d\tau \int_0^{2\pi\alpha'|p^+|} d\sigma \left[\frac{1}{2}\partial_+ X_I \partial_- X_I - \frac{1}{2}\mu^2 X_I^2 - i\bar{S}(\rho^A\partial_A - \mu\Pi)S\right] \tag{2.1}$$

where $\partial_\pm = \partial_\tau \pm \partial_\sigma$. The equations of motion following from the action (2.1) take the form

$$\partial_+\partial_- X^I + \mu^2 X^I = 0\,, \tag{2.2}$$

$$\partial_+ S^1 - \mu\Pi S^2 = 0\,, \qquad \partial_- S^2 + \mu\Pi S^1 = 0\,. \tag{2.3}$$

We use the following form for SO(8) gamma matrices

$$\gamma^I = \begin{pmatrix} 0 & \gamma^I_{a\dot{a}} \\ \tilde{\gamma}^I_{\dot{a}a} & 0 \end{pmatrix} \tag{2.4}$$

where $\tilde{\gamma}^I_{\dot{a}a} = (\gamma^{I^T})_{\dot{a}a}$ and take the SO(8) chirality matrix as

$$\gamma = \begin{pmatrix} 1_8 & 0 \\ 0 & -1_8 \end{pmatrix}. \tag{2.5}$$

In what follows, we assume that the spinors $S^A(\tau,\sigma)$, $A = 1, 2$, are positive chiral fermions, $\gamma S^A = S^A$, of the form

$$S^A_\alpha = \begin{pmatrix} S^A_a \\ 0 \end{pmatrix}, \tag{2.6}$$

where $\alpha = 1, \ldots, 16$ and $a = 1, \ldots, 8$.

Consider an open string attached on a Dp-brane in the plane wave background (1.1). The open string action is just defined by the action (2.1) with string length $\alpha = 2\alpha' p^+$ imposed with appropriate boundary conditions on each end of the open string.[1] For longitudinal coordinates X^r on D-branes without any worldvolume flux, we impose the Neumann boundary condition

$$\partial_\sigma X^r|_{\partial\Sigma} = 0\,, \tag{2.7}$$

[1]In this paper we will use the notation and the convention in [20] with more refined indices. Neumann coordinates X^r are decomposed into oblique directions $X^{\hat{r}}$ and usual parallel directions $X^{\dot{r}}$: $r = (\hat{r}, \dot{r})$. Similarly, Dirichlet coordinates $X^{r'}$ are also decomposed into oblique directions $X^{\hat{r}'}$ and usual parallel directions $X^{\dot{r}'}$: $r' = (\hat{r}', \dot{r}')$.

JHEP03(2004)058

while for transverse coordinates $X^{r'}$ we have the Dirichlet boundary condition

$$\partial_\tau X^{r'}|_{\partial\Sigma} = 0\,. \tag{2.8}$$

In the case to include gauge field excitations considered later, some Neumann boundary conditions have to be modified as follow [14, 22, 9, 18, 20]

$$(\partial_\sigma X^r \pm \mu X^r)|_{\partial\Sigma} = 0 \tag{2.9}$$

for some $r \in N$. The fermionic coordinates also have to satisfy the following boundary condition at each end of the open string [23]

$$(S^1 - \Omega S^2)|_{\partial\Sigma} = 0\,, \tag{2.10}$$

where the matrix Ω is the products of γ-matrices along worldvolume directions.

The boundary condition (2.10) has to be compatible with the fermionic equation of motion (2.3) and thus the possible type of D-branes shall be characterized by the matrix Γ defined by

$$\Gamma \equiv \Pi\Omega\Pi\Omega\,. \tag{2.11}$$

$D_\pm$-branes [12]–[20] are a specific class satisfying $\Gamma = \pm 1$. Since Ω is, in general, a basis in the Majorana representation of SO(8) Clifford algebra,[2] say, $\Omega^2 = \pm 1$ and $\Pi^2 = 1$, one can see that the matrix Γ satisfies the following properties:

$$\Pi\Omega\Pi\Omega = \Gamma = \Pi\Omega^T\Pi\Omega^T\,, \tag{2.13}$$

$$\Gamma\Gamma^T = 1\,, \qquad \Pi\Gamma\Pi\Gamma = 1\,. \tag{2.14}$$

Since the matrix Γ is also an element of SO(8) Clifford algebra, it must be either a symmetric or an antisymmetric matrix. In the case the matrix Γ is symmetric, i.e. $\Gamma^T = \Gamma$, it follows from (2.13) and (2.14) that

$$\Gamma^2 = 1\,, \qquad [\Pi, \Gamma] = 0 = [\Omega, \Gamma]\,. \tag{2.15}$$

The first equation in eq. (2.15) implies that Γ is a product of 0, 4 or 8 gamma matrices. Therefore we have three classes of D-brane in this case:

$$D_\pm\text{-brane}: \quad \Gamma = \pm 1\,, \tag{2.16}$$

$$ODp\text{-brane}: \quad \Gamma = \pm\gamma^{i_1 i_2 i_3' i_4'}\,, \qquad (p = 3, 5, 7)\,, \tag{2.17}$$

$$OD5\text{-brane}: \quad \Gamma = \pm\gamma\,. \tag{2.18}$$

[2] The isometry in the plane wave background (1.1) is indeed $\mathrm{SO}(4) \times \mathrm{SO}(4)' \times \mathbb{Z}_2$ where the $\mathbb{Z}_2$ symmetry interchanges simultaneously the two SO(4) directions [24]

$$\mathbb{Z}_2 : (x^1, x^2, x^3, x^4) \leftrightarrow (x^5, x^6, x^7, x^8)\,. \tag{2.12}$$

The open string theory on a D-brane is just defined by the closed string action (2.1) by imposing the boundary conditions, (2.7), (2.8), and (2.10). Thus the open string theory on a D-brane has to respect the symmetry of the closed string action which is $\mathrm{SO}(4) \times \mathrm{SO}(4)' \times \mathbb{Z}_2$. Then, we think, the gluing matrix Ω defining the boundary condition of open string fermions should be in the representation of $\mathrm{SO}(4) \times \mathrm{SO}(4)' \times \mathbb{Z}_2 \subset \mathrm{SO}(8)$ Clifford algebra rather than SO(8). This may explain why OD-branes are always at 45^o angle in oblique directions and the spectrum of D-branes is symmetric under the $\mathbb{Z}_2$ involution (2.12).

JHEP03(2004)058

D-brane type	Γ	Ω
$D_\pm$	± 1	$\Omega_{D_\pm}$
$OD3$	$\pm\gamma^{1256}$	$\frac{1}{2}(\gamma^1-\gamma^6)(\gamma^2\pm\gamma^5)$
$OD5$	$\pm\gamma^{1256}$	$\frac{1}{2}(\gamma^1-\gamma^6)(\gamma^2\mp\gamma^5)\gamma^{34}$ $\frac{1}{2}(\gamma^1-\gamma^6)(\gamma^2\mp\gamma^5)\gamma^{78}$ $\frac{1}{2}(\gamma^1-\gamma^6)(\gamma^2\pm\gamma^5)\gamma^{37}$
$OD7$	$\pm\gamma^{1256}$	$\frac{1}{2}(\gamma^1-\gamma^6)(\gamma^2\pm\gamma^5)\gamma^{3478}$
$OD_\pm 5$	$\pm\gamma$	$\frac{1}{4}(\gamma^1-\gamma^6)(\gamma^2\pm\gamma^5)(\gamma^3-\gamma^8)(\gamma^4+\gamma^7)$

Table 1: Flat D-branes with $\Gamma^2 = 1$.

The second equation in eq. (2.15) requires that the matrix Γ should contain an even number of gamma matrices in $\{\gamma^i,\ i = 1,\dots,4\}$ and $\{\gamma^{i'},\ i' = 5,\dots,8\}$. For example, $\Gamma = \pm\gamma^{1256}$ for ODp-branes in eq. (2.17). Then one can easily find the corresponding matrix Ω for the D-branes in eqs. (2.16)–(2.18). The solution shows that the D-branes in eqs. (2.16)–(2.18) contain 0, 2 and 4 oblique directions, respectively. Table 1 shows possible flat D-branes with particular polarizations. Other flat D-branes with different polarizations can be generated by $\mathrm{SO}(4) \times \mathrm{SO}(4)'$ rotations of the D-branes in table 1. The D-branes discussed in [9, 10] correspond to the $OD3$-brane with $\Gamma = -\gamma^{1256}$ and OD_-5-brane in table 1.

Applying the same argument in [10], one can show that the $OD_\pm 5$-branes in table 1 are obtained from the usual $D_\pm 5$-branes, respectively, by a special rotation R in the coset $\mathrm{SO}(8)/(\mathrm{SO}(4) \times \mathrm{SO}(4)')$ which leaves the matrix Γ invariant in the spinor space with positive chirality. Specifically, R describes a rotation by $\pi/4$ in each of the four planes $x^1 - x^6, x^2 - x^5, x^3 - x^8, x^4 - x^7$.

On the other hand, in the case the matrix Γ is antisymmetric, i.e. $\Gamma^T = -\Gamma$, it follows from (2.13) and (2.14) that

$$\Gamma^2 = -1\,, \qquad \{\Pi,\Gamma\} = 0 = \{\Omega,\Gamma\}\,. \tag{2.19}$$

The first equation in eq. (2.19) implies that Γ is a product of 2 or 6 gamma matrices. Thus we have two classes of oblique D-brane in this case:

$$ODp\text{-brane}: \ \ \Gamma = \pm\gamma^{i_1 i'_2}\,, \qquad (p = 3,5,7)\,, \tag{2.20}$$

$$OD5\text{-brane}: \ \ \Gamma = \pm\gamma^{i_1 i_2 i_3 i'_4 i'_5 i'_6}\,. \tag{2.21}$$

The second equation in eq. (2.19) forces the matrix Γ to contain an odd number of gamma matrices in $\{\gamma^i,\ i = 1,\dots,4\}$ and $\{\gamma^{i'},\ i' = 5,\dots,8\}$, for example, $\Gamma = \pm\gamma^{16}$ for ODp-branes in eq. (2.20). The matrix Ω for the D-branes in eqs. (2.20)–(2.21) then contain 1 and 3 oblique directions, respectively. A few solutions of Ω are, in this case, given by

$$\Omega = \frac{1}{\sqrt{2}}(\gamma^1 \mp \gamma^6)\gamma^2\,, \qquad (\Gamma = \pm\gamma^{16})\,, \tag{2.22}$$

$$\Omega = \frac{1}{2\sqrt{2}}(\gamma^1 \mp \gamma^6)(\gamma^2 + \gamma^5)(\gamma^3 - \gamma^8)\gamma^4\,, \qquad (\Gamma = \pm\gamma^{123568})\,. \tag{2.23}$$

3. Open string mode expansion for D-branes

According to the gluing matrix Ω in table 1 and in eqs. (2.22)–(2.23), we define diagonal coordinates

$$X^{\hat{r}} = \frac{1}{\sqrt{2}}(X^r \pm X^{r'})\,, \qquad X^{\hat{r}'} = \frac{1}{\sqrt{2}}(X^{r'} \mp X^r) \tag{3.1}$$

with the index notation explained in footnote 1. For an OD5-brane described by $\Omega = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 - \gamma^5)\gamma^{34}$, for example, we have

$$\begin{aligned}
\text{Neumann}: \ X^{\hat{1}} &= \frac{1}{\sqrt{2}}(X^1 - X^6)\,, \quad X^{\hat{2}} = \frac{1}{\sqrt{2}}(X^2 - X^5)\,, \quad X^{\dot{3}} = X^3\,, \quad X^{\dot{4}} = X^4\,,\\
\text{Dirichlet}: X^{\hat{5}'} &= \frac{1}{\sqrt{2}}(X^5 + X^2), \quad X^{\hat{6}'} = \frac{1}{\sqrt{2}}(X^6 + X^1), \quad X^{\dot{7}'} = X^7\,, \quad X^{\dot{8}'} = X^8\,.
\end{aligned}$$

Since the bosonic equation of motion (2.2) is invariant under the coordinate redefinition (3.1) and insensitive to the fermionic boundary condition (2.10), the mode expansion satisfying the Neumann or the Dirichlet boundary condition, eqs. (2.7)–(2.9), is exactly the same as the usual $D_\pm$-branes. See for this, for example, ref. [20].

The mode expansion of the spinor field is found to be

$$\begin{aligned}
S^1(\tau,\sigma) &= S^1_0(\tau,\sigma) + \sum_{n\neq 0} c_n(\varphi^1_n(\tau,\sigma)\Omega\widetilde{S}_n + i\rho_n\varphi^2_n(\tau,\sigma)\Pi S_n)\,,\\
S^2(\tau,\sigma) &= S^2_0(\tau,\sigma) + \sum_{n\neq 0} c_n(\varphi^2_n(\tau,\sigma)S_n - i\rho_n\varphi^1_n(\tau,\sigma)\Pi\Omega\widetilde{S}_n)\,,
\end{aligned} \tag{3.2}$$

where the basis functions $\varphi^{1,2}_n(\tau,\sigma)$ are defined by

$$\varphi^1_n(\tau,\sigma) = e^{-i(\omega_n\tau - \frac{n}{|\alpha|}\sigma)}\,, \qquad \varphi^2_n(\tau,\sigma) = e^{-i(\omega_n\tau + \frac{n}{|\alpha|}\sigma)} \tag{3.3}$$

and

$$\omega_n = \mathrm{sign}(n)\sqrt{\mu^2 + \frac{n^2}{\alpha^2}}\,, \qquad \rho_n = \frac{\omega_n - n/|\alpha|}{\mu}\,, \qquad c_n = \frac{1}{\sqrt{1+\rho_n^2}}\,. \tag{3.4}$$

$S^A_0(\tau,\sigma)$ in eq. (3.2) are possible zero modes to be fixed later and the modes $\widetilde{S}_n$ are determined by requiring the boundary condition (2.10) with the gluing matrix Ω satisfying the relation (2.11):

$$(1+\rho_n^2\Gamma)\widetilde{S}_n = \Big((1-\rho_n^2) - i\rho_n\Pi\Omega(1+\Gamma^T)\Big)S_n\,. \tag{3.5}$$

In the case of $\Gamma^T = \Gamma$, it is useful to decompose the spinors $S^A(\tau,\sigma)$ into eigenspinors of Γ by defining

$$S^A_\pm(\tau,\sigma) = P_\pm S^A(\tau,\sigma)\,, \tag{3.6}$$

where

$$P_\pm = \frac{1}{2}(1\pm\Gamma)\,. \tag{3.7}$$

It follows from eq. (2.15) that the equations of motion for the spinors $S_\pm^A(\tau,\sigma)$, eq. (2.3), are completely separated into two independent equations of motion

$$\partial_+ S_+^1 - \mu\Pi S_+^2 = 0\,, \qquad \partial_- S_+^2 + \mu\Pi S_+^1 = 0\,, \tag{3.8}$$
$$\partial_+ S_-^1 - \mu\Pi S_-^2 = 0\,, \qquad \partial_- S_-^2 + \mu\Pi S_-^1 = 0 \tag{3.9}$$

and the boundary condition, eq. (2.10), can be separately imposed for the spinors $S_\pm^A(\tau,\sigma)$

$$(S_+^1 - \Omega S_+^2)|_{\partial\Sigma} = 0\,, \tag{3.10}$$
$$(S_-^1 - \Omega S_-^2)|_{\partial\Sigma} = 0\,. \tag{3.11}$$

Then one can immediately see from eqs. (3.2) and (3.5) that the spinor $S_+^A(\tau,\sigma)$ has a D_+-like mode expansion while $S_-^A(\tau,\sigma)$ does a D_--like mode expansion [20]. This should be obvious since $\Gamma = +1$ in the space spanned by S_+^A while $\Gamma = -1$ in the space spanned by S_-^A, namely,

$$\Gamma S_\pm^A(\tau,\sigma) = \pm S_\pm^A(\tau,\sigma)\,. \tag{3.12}$$

Based on this observation, we can easily find the zero modes $S_0^A(\tau,\sigma) = S_{-0}^A(\tau,\sigma) + S_{+0}^A(\tau,\sigma)$ in eq. (3.2):

$$\begin{aligned}
S_0^1(\tau,\sigma) &= \cos\mu\tau S_0^- - \sin\mu\tau\Omega\Pi S_0^- + \cosh\mu\sigma S_0^+ + \sinh\mu\sigma\Omega\Pi S_0^+\,,\\
S_0^2(\tau,\sigma) &= \cos\mu\tau\Omega^T S_0^- - \sin\mu\tau\Pi S_0^- + \cosh\mu\sigma\Omega^T S_0^+ + \sinh\mu\sigma\Pi S_0^+\,,
\end{aligned} \tag{3.13}$$

where

$$P_\pm S_0^\pm = S_0^\pm\,, \qquad P_\mp S_0^\pm = 0\,. \tag{3.14}$$

The commutation relations between the modes read as

$$\begin{aligned}
\{S_0^{-a}, S_0^{-b}\} &= \frac{1}{4}P_-^{ab}\,,\\
\{S_0^{+a}, S_0^{+b}\} &= \frac{\pi\mu|\alpha|}{4\sinh\pi\mu|\alpha|}\Big(P_+^{ab}\cosh\pi\mu|\alpha| - (P_+\Omega\Pi)^{ab}\sinh\pi\mu|\alpha|\Big)\,,\\
\{S_n^{\pm a}, S_m^{\pm b}\} &= \frac{1}{4}\delta_{n+m,0}P_\pm^{ab}\,,
\end{aligned} \tag{3.15}$$

where $S_n^\pm = P_\pm S_n$ for $n \neq 0$.

One can show that there is no spinor zero mode in the case of $\Gamma^T = -\Gamma$, that is, $S_0^A(\tau,\sigma) = 0$. This fact signals no kinematical supersymmetry in this case. In section 5 we will indeed show that the OD-branes in this case, eqs. (2.20)–(2.21), preserve no supersymmetry. Thus we will not give a detailed analysis for this class of OD-branes.

4. Intersecting D-branes

In this section, we will generalize the previous analysis to the case of intersecting D-branes using the formalism in [21]. That is, we now consider an open string stretched between Dp-brane and Dq-brane with appropriate boundary conditions on each end of the open string in the plane wave background (1.1). In particular, the fermionic coordinates have to satisfy the following boundary condition at each end of the open string

$$(S^1 - \Omega_0 S^2)|_{\sigma=0} = 0\,, \qquad (S^1 - \Omega_\pi S^2)|_{\sigma=\pi\alpha} = 0\,, \tag{4.1}$$

with the matrix $\Omega_\theta = (\Omega_0, \Omega_\pi)$ satisfying

$$\Pi\Omega_0\Pi\Omega_0 = \Gamma_0\,, \qquad \Pi\Omega_\pi\Pi\Omega_\pi = \Gamma_\pi\,. \tag{4.2}$$

Here the D-brane is either a $D_\pm$-brane or an OD-brane.

The coordinates $X^I(\tau,\sigma)$ of a $p-q$ string can be partitioned into four sets, NN, DD, ND, and DN, according to whether the coordinate X^I has Neumann (N) or Dirichlet (D) boundary condition at each end.[3] For the same reason in section 3, the bosonic coordinates have the same mode expansion as the case of $D_\pm$-brane intersection.

The mode expansion of the spinor field can be determined by exactly the same method as that in [21]. We take an appropriate combination of spinor fields $\xi^A(\tau,\sigma)$ with integer modes and $\eta^A(\tau,\sigma)$ with half-integer modes or with $\mathbf{R}$-modes to be compatible with supersymmetry:

$$\begin{aligned} S^1(\tau,\sigma) &= \begin{cases} I_+\xi^1(\tau,\sigma) + I_-\eta^1(\tau,\sigma)\,, & \text{for A-type}\,; \\ I_-\xi^1(\tau,\sigma) + I_+\eta^1(\tau,\sigma)\,, & \text{for B-type}\,, \end{cases} \\ S^2(\tau,\sigma) &= I_+\xi^2(\tau,\sigma) + I_-\eta^2(\tau,\sigma)\,, \end{aligned} \tag{4.3}$$

where 16×16 matrices I_+ and I_- are defined by

$$I_+ = \frac{1}{2}(1 + \Omega_0^T\Omega_\pi)\,, \qquad I_- = \frac{1}{2}(1 - \Omega_0^T\Omega_\pi)\,. \tag{4.4}$$

In eq. (4.3), the A-type solution is for $|p-q| = 0,4,8$ in $Dp - Dq$ brane configurations while the B-type solution for $|p-q| = 2,6$. The spinors $\xi^A(\tau,\sigma)$ and $\eta^A(\tau,\sigma)$ are taken as the solution of the equations of motion (2.3) satisfying the boundary condition (4.1) at $\sigma = 0$:

$$\begin{aligned} \xi^1(\tau,\sigma) &= S_0^1(\tau,\sigma) + \sum_{n\neq0} c_n\big(\varphi_n^1(\tau,\sigma)\Omega_0\widetilde{S}_n + i\rho_n\varphi_n^2(\tau,\sigma)\Pi S_n\big)\,, \\ \xi^2(\tau,\sigma) &= S_0^2(\tau,\sigma) + \sum_{n\neq0} c_n\big(\varphi_n^2(\tau,\sigma)S_n - i\rho_n\varphi_n^1(\tau,\sigma)\Pi\Omega_0\widetilde{S}_n\big)\,, \\ \eta^1(\tau,\sigma) &= \sum_\kappa c_\kappa\big(\varphi_\kappa^1(\tau,\sigma)\Omega_0\widetilde{S}_\kappa + i\rho_\kappa\varphi_\kappa^2(\tau,\sigma)\Pi S_\kappa\big)\,, \\ \eta^2(\tau,\sigma) &= \sum_\kappa c_\kappa\big(\varphi_\kappa^2(\tau,\sigma)S_\kappa - i\rho_\kappa\varphi_\kappa^1(\tau,\sigma)\Pi\Omega_0\widetilde{S}_\kappa\big)\,, \end{aligned} \tag{4.5}$$

[3]For intersecting D-branes, we will use indices $(r,s,\cdots) = (\hat{r},\hat{s},\cdots;\dot{r},\dot{s},\cdots)$ $(r',s',\cdots) = (\hat{r}',\hat{s}',\cdots;\dot{r}',\dot{s}',\cdots)$, $(i,j,\cdots) = (\hat{i},\hat{j},\cdots;\dot{i},\dot{j},\cdots)$, and $(i',j',\cdots) = (\hat{i}',\hat{j}',\cdots;\dot{i}',\dot{j}',\cdots)$ for NN, DD, ND, and DN coordinates, respectively, with a distinction between hatted indices for oblique directions and dotted indices for parallel directions.

JHEP03(2004)058

where the basis functions $\varphi_\nu^{1,2}(\tau,\sigma)$ are defined by

$$\varphi_\nu^1(\tau,\sigma) = e^{-i(\omega_\nu\tau - \frac{\nu}{|\alpha|}\sigma)}\,, \qquad \varphi_\nu^2(\tau,\sigma) = e^{-i(\omega_\nu\tau + \frac{\nu}{|\alpha|}\sigma)} \tag{4.6}$$

and

$$\omega_\nu = \mathrm{sign}(\nu)\sqrt{\mu^2 + \frac{\nu^2}{\alpha^2}}\,, \qquad \rho_\nu = \frac{\omega_\nu - \nu/|\alpha|}{\mu}\,, \qquad c_\nu = \frac{1}{\sqrt{1+\rho_\nu^2}} \tag{4.7}$$

for $\nu = n \in \mathbb{Z}$ or $\kappa \in \mathbb{Z} + \frac{1}{2}, \mathbf{R}$. Here $S_0^A(\tau,\sigma)$ in the case of $\Gamma_0^T = \Gamma_0$ are given by eq. (3.13) with a replacement $(\Omega,\Gamma) \to (\Omega_0,\Gamma_0)$ while $S_0^A(\tau,\sigma) = 0$ in the case of $\Gamma_0^T = -\Gamma_0$ and

$$(1+\rho_\nu^2\Gamma_0)\widetilde{S}_\nu = \Big((1-\rho_\nu^2) - i\rho_\nu\Pi\Omega_0(1+\Gamma_0^T)\Big)S_\nu\,. \tag{4.8}$$

We now require the spinors $S^A(\tau,\sigma)$ in eq. (4.3) to satisfy the equations of motion (2.3) and then we need the following condition on $I_\pm$:

$$\Pi I_\pm = \begin{cases} I_\pm\Pi\,, & \text{for A-type}\,; \\ I_\mp\Pi\,, & \text{for B-type.} \end{cases} \tag{4.9}$$

Noting that the matrices in eq. (4.9) are acting on the positive chirality spinors $S^A(\tau,\sigma)$, one can see that the condition (4.9) is equivalent to the following constraint

$$\Gamma_0\Gamma_\pi = \begin{cases} 1 \text{ or } \gamma\,, & \text{for } \Gamma_\theta^T = \Gamma_\theta\,; \\ -1 \text{ or } -\gamma\,, & \text{for } \Gamma_\theta^T = -\Gamma_\theta\,. \end{cases} \tag{4.10}$$

The condition (4.10) clearly explains why a D_--brane cannot have a supersymmetric intersection with a D_+-brane, as was shown in [21], since $\Gamma_0 = -1$ and $\Gamma_\pi = 1$ for this kind of intersection. In addition, the condition (4.10) implies that there may be a supersymmetric intersection between different classes of OD-brane or an $OD_\pm5$-brane and a $D_\pm p$-brane only if they satisfy $\Gamma_0\Gamma_\pi = \gamma$. In section 6 we will show that this case preserves only kinematical supersymmetries.

Note that $\Omega_\theta^T = -\Omega_\theta$ for $D3$- and $D7$-branes, but $\Omega_\theta^T = \Omega_\theta$ for $D5$-branes and thus

$$\Omega_0^T\Omega_\pi = \begin{cases} \Omega_0\Omega_\pi^T\,, & \text{for A-type}\,; \\ -\Omega_0\Omega_\pi^T, & \text{for B-type}\,. \end{cases} \tag{4.11}$$

Using eq. (4.11), we get useful identities [21]:

$$\Omega_\theta I_\pm = I_\pm\Omega_\theta\,, \qquad I_\pm\Omega_0 = \pm I_\pm\Omega_\pi\,, \qquad \text{for A-type}\,; \tag{4.12}$$

$$\Omega_\theta I_\pm = I_\mp\Omega_\theta\,, \qquad I_\pm\Omega_0 = \mp I_\pm\Omega_\pi\,, \qquad \text{for B-type}\,. \tag{4.13}$$

It is not difficult to check using eqs. (4.12)–(4.13) that the spinors in eq. (4.3) satisfy the boundary conditions (4.1) only if the mode number κ satisfies the following equation

$$e^{2\pi i\kappa}\Big((1-\rho_\kappa^2) - i\rho_\kappa\Pi\Omega_0(1+\Gamma_0^T)\Big)S_\kappa = -\Big((1-\rho_\kappa^2) + i\rho_\kappa\Pi\Omega_0(1+\Gamma_0^T)\Big)S_\kappa\,. \tag{4.14}$$

For example, $\kappa \in \mathbb{Z} + \frac{1}{2}$ when $\Gamma_0 = -1$ and when $\Gamma_0 = 1$ [21]

$$e^{2\pi i\kappa} = -\frac{\kappa + i\mu|\alpha|}{\kappa - i\mu|\alpha|} \qquad \text{or} \qquad -\frac{\kappa - i\mu|\alpha|}{\kappa + i\mu|\alpha|}\,. \tag{4.15}$$

The matrix $\Omega_0^T\Omega_\pi$ consists of products of γ-matrices along the ND and DN directions. Since $(\Omega_0^T\Omega_\pi)^2 = 1$ and $\mathrm{Tr}(\Omega_0^T\Omega_\pi) = 0$ for $\Omega_0^T\Omega_\pi \neq \pm 1$, there can be only three kinds of possibility:

$$\Omega_0^T\Omega_\pi = \begin{cases} \pm 1\,, & \sharp_{ND} = 0\,, \\ \pm\gamma\,, & \sharp_{ND} = 8, \\ \pm\begin{pmatrix} \Xi & 0 \\ 0 & \pm\Xi \end{pmatrix}, & \sharp_{ND} = 4\,, \end{cases} \tag{4.16}$$

where

$$\Xi = \begin{pmatrix} 1_4 & 0 \\ 0 & -1_4 \end{pmatrix}, \tag{4.17}$$

and $\sharp_{ND}$ denotes the total number of ND and DN directions.

The case $\Omega_0^T\Omega_\pi = 1$ corresponds to parallel Dp-branes while the case $\Omega_0^T\Omega_\pi = -1$ corresponds to Dp-anti-Dp branes, but the cases $\Omega_0^T\Omega_\pi = \pm\gamma$ and $\Omega_0^T\Omega_\pi = \pm\Xi$ correspond to $Dp - Dq$ or Dp-anti-Dq branes with $\sharp_{ND} = 8$ and $\sharp_{ND} = 4$, respectively. Note that the B-type branes allow only the $\sharp_{ND} = 4$ case.

5. Supersymmetry of flat D-branes

In a light-cone gauge, the 32 components of the supersymmetries for a closed string decompose into kinematical supercharges, Q_a^{+A}, and dynamical supercharges, $Q_{\dot{a}}^{-A}$. For a closed superstring in the plane wave background with the action (2.1), the conserved super-Nöther charges were identified by Metsaev [6]:

$$Q^{+1} = \frac{\sqrt{2p^+}}{2\pi\alpha' p^+}\int_0^{2\pi\alpha'|p^+|} d\sigma(\cos\mu\tau S^1 - \sin\mu\tau\Pi S^2)\,, \tag{5.1}$$

$$Q^{+2} = \frac{\sqrt{2p^+}}{2\pi\alpha' p^+}\int_0^{2\pi\alpha'|p^+|} d\sigma(\cos\mu\tau S^2 + \sin\mu\tau\Pi S^1)\,, \tag{5.2}$$

$$\sqrt{2p^+}Q^{-1} = \frac{1}{2\pi\alpha' p^+}\int_0^{2\pi\alpha'|p^+|} d\sigma\Big(\partial_- X^I\gamma^I S^1 - \mu X^I\gamma^I\Pi S^2\Big)\,, \tag{5.3}$$

$$\sqrt{2p^+}Q^{-2} = \frac{1}{2\pi\alpha' p^+}\int_0^{2\pi\alpha'|p^+|} d\sigma\Big(\partial_+ X^I\gamma^I S^2 + \mu X^I\gamma^I\Pi S^1\Big)\,. \tag{5.4}$$

The kinematical supersymmetry is, in general, related to a shift of spinor fields and thus generated by spinor zero modes. We showed in eq. (3.13) that there are two kinds of spinor zero modes, $S_0^\pm$, when $\Gamma^T = \Gamma$. Therefore we expect that there are two kinds of kinematical supersymmetry in this case where each of supersymmetry is generated by $S_0^\pm$. Indeed we will show that an open string on a D-brane with even number of oblique directions preserves 8 kinematical supersymmetries of which 4 supersymmetries generated by S_0^- are descending from the closed string and another 4 supersymmetries generated by S_0^+ are the new kind of supersymmetry, not descending from the closed string, as identified by Skenderis and Taylor [17] for D_+-branes.

Indeed it was shown in [17, 19] that the light-cone action admits an infinite number of worldsheet symmetries and they all act on the open string spectrum. Since the worldsheet theory on OD-branes can be completely separable into $D_{\pm}$-like ones as was shown in eqs. (3.8)–(3.11), it is obvious that the worldsheet symmetries in [17, 19] are also applicable to the OD-branes. Since we are interested in the open string supersymmetry, we will focus only on the kinematical supersymmetry generated by spinor zero modes S_0^+.

To see this, let us first consider the kinematical supersymmetry descending from the closed string

$$q_-^+ = Q^{+1} + \Omega Q^{+2} \tag{5.5}$$

from which the Nöther charge density $q_{-\tau}^+$ reads as

$$q_{-\tau}^+ = \sqrt{2p^+}\Big(\cos\mu\tau(S^1 + \Omega S^2) + \sin\mu\tau\Omega\Pi(S^1 - \Gamma\Omega S^2)\Big)\,. \tag{5.6}$$

It is easy to show using the equations of motion, eqs. (2.2) and (2.3), that the kinematical supercharge density $q_{-\tau}^+$ satisfies the following conservation law

$$\frac{\partial q_{-\tau}^+}{\partial\tau} + \frac{\partial q_{-\sigma}^+}{\partial\sigma} = 0 \tag{5.7}$$

with

$$q_{-\sigma}^+ = \sqrt{2p^+}\Big(\cos\mu\tau(S^1 - \Omega S^2) + \sin\mu\tau\Omega\Pi(S^1 + \Gamma\Omega S^2)\Big)\,. \tag{5.8}$$

One can immediately see from eqs. (5.8) and (3.12) that the kinematical supersymmetry defined by

$$q_{D_-}^+ = P_- q_-^+ \tag{5.9}$$

is strictly preserved and is generated by the zero mode S_0^-.

Next consider the kinematical supersymmetry, not descending from the closed string, preserved on D_+-branes

$$q_+^+ = \frac{1}{\pi|\alpha|}\int_0^{\pi|\alpha|} d\sigma q_{+\tau}^+ \tag{5.10}$$

where

$$q_{+\tau}^+ = \sqrt{2p^+}\sqrt{\frac{\pi\mu|\alpha|}{\sinh\pi\mu|\alpha|}}\, e^{\mu(\sigma - \frac{1}{2}\pi|\alpha|)\Omega\Pi}(S^1 + \Omega S^2)\,. \tag{5.11}$$

It is also easy to show that

$$\begin{aligned}\frac{\partial q_{+\tau}^+}{\partial\tau} + \frac{\partial q_{+\sigma}^+}{\partial\sigma} &= \mu\sqrt{2p^+}\sqrt{\frac{\pi\mu|\alpha|}{\sinh\pi\mu|\alpha|}}\, e^{\mu(\sigma - \frac{1}{2}\pi|\alpha|)\Omega\Pi}\Omega\Pi \times \\ &\times\Big((S^1 - \Omega S^2) - (S^1 - \Gamma\Omega S^2)\Big)\end{aligned} \tag{5.12}$$

where

$$q_{+\sigma}^+ = \sqrt{2p^+}\sqrt{\frac{\pi\mu|\alpha|}{\sinh\pi\mu|\alpha|}}\, e^{\mu(\sigma - \frac{1}{2}\pi|\alpha|)\Omega\Pi}(S^1 - \Omega S^2)\,. \tag{5.13}$$

D-brane type	Γ	Ω	$q^+_{D_-}$	$q^+_{D_+}$	q^-
$\mathbf{D_-5}$	-1	$(3,1),\ (1,3)$	8	0	8
$\mathbf{D_+(2n+1)}$	1	$(n,n),\quad n=1,2,3,4$	0	8	4
$OD3$	$\pm\gamma^{1256}$	$\frac{1}{2}(\gamma^1-\gamma^6)(\gamma^2\pm\gamma^5)$	4	4	4
$\mathbf{OD5}$	$\pm\gamma^{1256}$	$\frac{1}{2}(\gamma^1-\gamma^6)(\gamma^2\mp\gamma^5)\gamma^{34}$ $\frac{1}{2}(\gamma^1-\gamma^6)(\gamma^2\mp\gamma^5)\gamma^{78}$	4	4	4
$\mathbf{OD5}$	$\pm\gamma^{1256}$	$\frac{1}{2}(\gamma^1-\gamma^6)(\gamma^2\pm\gamma^5)\gamma^{37}$	4	4	2
$\mathbf{OD7}$	$\pm\gamma^{1256}$	$\frac{1}{2}(\gamma^1-\gamma^6)(\gamma^2\pm\gamma^5)\gamma^{3478}$	4	4	2
OD_+5	γ	$\frac{1}{4}(\gamma^1-\gamma^6)(\gamma^2+\gamma^5)(\gamma^3-\gamma^8)(\gamma^4+\gamma^7)$	0	8	0
OD_-5	$-\gamma$	$\frac{1}{4}(\gamma^1-\gamma^6)(\gamma^2-\gamma^5)(\gamma^3-\gamma^8)(\gamma^4+\gamma^7)$	8	0	8

Table 2: Supersymmetry of flat D-branes. A D-brane with a gauge field condensate is denoted by the boldface. $q^+_{D_-}$ ($q^+_{D_+}$) is the number of unbroken kinematical supersymmetry of D_--type (D_+-type).

It is thus obvious that the kinematical supersymmetry defined by

$$q^+_{D_+} = P_+ q^+_+ \tag{5.14}$$

is preserved and is generated by the zero mode S^+_0.

In consequence, an open string on a D-brane satisfying $\Gamma^T = \Gamma$ preserves 8 kinematical supersymmetries of which 4 supersymmetries are generated by S^-_0 and another 4 supersymmetries are generated by S^+_0. We summarized our results on the supersymmetry of D-branes in table 2.

On the other hand, an open string on a D-brane satisfying $\Gamma^T = -\Gamma$ preserves no kinematical supersymmetry since the matrix Γ has eigenvalues $\pm i$ in this case and thus $q^+_{-\sigma}|_{\partial\Sigma}$ in eq. (5.8) and the right hand side of eq. (5.12) cannot vanish for any component of spinors. This is consistent with the fact that there is no spinor zero mode when $\Gamma^T = -\Gamma$.

Now we investigate the dynamical supersymmetry preserved by an open string on a D-brane characterized by Γ in eq. (2.11). The dynamical supercharge of an open string is given by a combination of those of a closed string compatible with the open string boundary conditions. Due to the boundary condition (2.10), it turns out that the conserved dynamical supercharge is given by (a subset of)[4]

$$q^- = Q^{-1} - \Omega Q^{-2}\,. \tag{5.15}$$

Using the similar recipe used in the kinematical supersymmetry, it is not difficult to show that the dynamical supercharge density q^-_τ in eq. (5.15) also satisfies the conservation law

$$\frac{\partial q^-_\tau}{\partial\tau} + \frac{\partial q^-_\sigma}{\partial\sigma} = 0\,, \tag{5.16}$$

[4]Note that the current definition of the dynamical supercharge q^- differs by a factor $-\Omega$ from that in [20, 21].

where

$$q_\sigma^- = \sqrt{\frac{1}{2p^+}}\Big((\partial_\tau X^r\gamma^r - \partial_\sigma X^{r'}\gamma^{r'})(S^1-\Omega S^2) + (\partial_\tau X^{r'}\gamma^{r'} - \partial_\sigma X^r\gamma^r)(S^1+\Omega S^2) + \\ + \mu X^r\gamma^r\Omega\Pi(S^1+\Gamma\Omega S^2) - \mu X^{r'}\gamma^{r'}\Omega\Pi(S^1-\Gamma\Omega S^2)\Big)\,. \qquad (5.17)$$

If $\Gamma=\pm1$, we definitely recover the $D_\pm$-brane case [20]. It was shown in [14] that D_--branes of type $(+,-,3,1)$ or $(+,-,1,3)$ with a constant worldvolume flux also preserve 16 supersymmetries whose possibility was not discussed in [20]. Here we will show how it can be true. In the presence of a constant worldvolume flux F_{+r}, there is an additional boundary action given by

$$S_B = \frac{1}{2\pi\alpha' p^+}\int d\tau \int_0^{2\pi\alpha'|p^+|} d\sigma F_{+r}\partial_\tau X^+\partial_\sigma X^r = \frac{F_{+r}}{2\pi\alpha' p^+}\int_{\partial\Sigma} d\tau X^r\,. \qquad (5.18)$$

This boundary term affects the Neumann boundary condition as follows

$$\partial_\sigma X_r - F_{+r} = 0\,. \qquad (5.19)$$

With this boundary condition, only $D5$-brane among D_--branes can preserve dynamical supersymmetry. To see this, let us take a $(+,-,3,1)$-brane, for definiteness, extended along $(+,-,1,2,3,5)$ directions, say, $N=(1,2,3,5)$ and $D=(4,6,7,8)$ and thus we have $\Gamma=-1$ and $\Omega\Pi=-\gamma^{45}$ in this case. If we turn on the flux $F_{+5}=\mu q$ only, otherwise $F_{+r}=0$, the current q_σ^- in eq. (5.17) at the boundary reduces to

$$q_\sigma^-|_{\partial\Sigma} = -\sqrt{\frac{1}{2p^+}}(\partial_\sigma X^5 - \mu X^4)\gamma^5(S^1+\Omega S^2)\,, \qquad (5.20)$$

where we used $\partial_\sigma X^r|_{\partial\Sigma}=0,\ r=1,2,3$ and $X^{r'}|_{\partial\Sigma}=0,\ r'=6,7,8$. Thus we see that $q_\sigma^-|_{\partial\Sigma}$ identically vanishes if $X^4|_{\partial\Sigma}=q$, as was shown in [14]. From the above analysis, we see that the condition $\gamma^r=\pm\gamma^{r'}\Omega\Pi$ is necessary to preserve the dynamical supersymmetry with a constant flux. Obviously $(+,-,3,1)$- and $(+,-,1,3)$-branes only satisfy this condition.

When $\Gamma=\Omega\Pi\Omega\Pi=1$, there is also a new possibility for D_+-branes of type $(+,-,n,n)$ with $n=1,2,3,4$ to preserve dynamical supersymmetries by introducing a gauge field excitation,[5] whose possibility was anticipated by Hikida and Yamaguchi [9] from general supersymmetry arguments. To prove this claim, we first introduce projection matrices defined by

$$P_\pm^{D+} = \frac{1}{2}(1\pm\Omega\Pi) \qquad (5.21)$$

and projected supercharges

$$q_\pm^- \equiv P_\pm^{D+}(Q^{-1}-\Omega Q^{-2})\,. \qquad (5.22)$$

[5]This idea was arisen from a discussion with Yasuaki Hikida who we thank for the helpful discussion.

To proceed our argument, it is convenient to decompose Neumann and Dirichlet coordinates into two SO(4) directions: $r = (r_1, r_2)$ and $r' = (r'_1, r'_2)$ where the subscripts $1 \in \mathrm{SO}(4)$ and $2 \in \mathrm{SO}(4)'$ are used for that purpose. From the definition (5.21), we get

$$[P_{\pm}^{D+}, \gamma^{r_1}] = 0 = [P_{\pm}^{D+}, \gamma^{r'_2}]\,, \tag{5.23}$$

$$P_{\pm}^{D+}\gamma^{r_2} = \gamma^{r_2} P_{\mp}^{D+}\,, \qquad P_{\pm}^{D+}\gamma^{r'_1} = \gamma^{r'_1} P_{\mp}^{D+}\,. \tag{5.24}$$

From eq. (5.17) using eqs. (5.23)–(5.24), it is easy to find the condition for the projected supercharge (5.22) to be conserved since $q^-_{\pm\sigma}$ at the worldsheet boundary $\partial\Sigma$ reduces to

$$\begin{aligned} q^-_{\pm\sigma}\Big|_{\partial\Sigma} = \sqrt{\frac{1}{2p^+}}\Big((\partial_\tau X^{r'_2}\gamma^{r'_2} - \partial_\sigma X^{r_1}\gamma^{r_1} \pm \mu X^{r_1}\gamma^{r_1})P_{\pm}^{D+}(S^1 + \Omega S^2) + \\ + (\partial_\tau X^{r'_1}\gamma^{r'_1} - \partial_\sigma X^{r_2}\gamma^{r_2} \mp \mu X^{r_2}\gamma^{r_2})P_{\mp}^{D+}(S^1 + \Omega S^2)\Big)_{\partial\Sigma}. \end{aligned} \tag{5.25}$$

We see that the Neumann boundary condition should be modified as follows

$$\Big(\partial_\sigma X^{r_1} \mp \mu X^{r_1}\Big)_{\partial\Sigma} = 0 = \Big(\partial_\sigma X^{r_2} \pm \mu X^{r_2}\Big)_{\partial\Sigma} \tag{5.26}$$

to preserve the dynamical supersymmetry. This kind of boundary condition can be realized by introducing a gauge field excitation of the form

$$S_B = \frac{1}{2\pi\alpha' p^+}\int d\tau \int_0^{2\pi\alpha'|p^+|} d\sigma F^{-r}\partial_\sigma X^r = \frac{\mu}{4\pi\alpha' p^+}\int_{\partial\Sigma} d\tau(\pm X^{r_1}X^{r_1} \mp X^{r_2}X^{r_2})\,, \tag{5.27}$$

where the Born-Infeld flux F^{-r} satisfies

$$\partial_r F^{-r} = 0\,, \qquad r = (r_1, r_2)\,. \tag{5.28}$$

Therefore we proved that D_+-branes of type $(+,-,n,n)$ with $n = 1,2,3,4$ preserve 4 dynamical supersymmetries by introducing a gauge field excitation, consistent with the result in [9]. Note that the dynamical supersymmetry in this case is preserved regardless of transverse locations of D-brane. One can check that eq. (5.28) is just the equation of motion for the gauge field A, eq. (A.2) with $F_5 = 0$. In this case we don't need an additional Chern-Simons coupling such as (A.1).

When $\Gamma^T = -\Gamma$, however, eq. (5.17) immediately shows that there is no chance for $q^-_\sigma|_{\partial\Sigma}$ to vanish and thus an open string on this D-brane does not preserve any dynamical supersymmetry at all.

In the case of $\Gamma^T = \Gamma$, on the other hand, the condition for $q^-_\sigma|_{\partial\Sigma}$ to vanish depends on the eigenvalue of the matrix Γ. We therefore introduce projected supercharges defined by

$$q^-_{\pm} \equiv P_{\pm}(Q^{-1} - \Omega Q^{-2})\,. \tag{5.29}$$

In particular, when $\Gamma = -\gamma$, eight q^-_+ supersymmetries survive only if $X^{\hat{r}'} = 0$ for all $\hat{r}' \in D$ while q^-_- identically vanish because $[\gamma, \Omega] = [\gamma, \Pi] = 0$ and $\gamma S^A = S^A$. This might be expected since the $OD_{\pm}5$-branes can be related to the ordinary $D_{\pm}5$-branes by a special rotation R in the coset $\mathrm{SO}(8)/(\mathrm{SO}(4)\times\mathrm{SO}(4)')$ which leaves the matrix Γ invariant in the spinor space with positive chirality [10].

One may wonder whether a D_+5-brane with flux, preserving 8 dynamical supersymmetries, can be rotated to an OD_+5-brane preserving the same kind of supersymmetry. In this case the Neumann boundary condition of the OD_+5-brane needs to be modified to $\partial_\sigma X^r - \mu X^r = 0$, $\forall r \in N$. The general expression (5.17), however, implies that it is not possible since we need $\Omega\Pi S^A = S^A$ for $OD5$-branes, which is never satisfied. It will be shown in appendix A that this can also be understood by investigating a worldvolume Chern-Simons coupling.

To find the condition for the projected supercharge to be conserved, first note that

$$P_\pm \gamma^{\hat{r}} = \gamma^{\hat{r}} P_\mp \,, \qquad P_\pm \gamma^{\hat{r}'} = \gamma^{\hat{r}'} P_\mp \,, \tag{5.30}$$

$$[P_\pm, \gamma^{\dot{r}}] = 0 = [P_\pm, \gamma^{\dot{r}'}] \,. \tag{5.31}$$

It is then easy to read off the value of $q^-_{\pm\sigma}|_{\partial\Sigma}$ from eq. (5.17) using eqs. (5.30)–(5.31):

$$q^-_{+\sigma}\Big|_{\partial\Sigma} = -\sqrt{\frac{1}{2p^+}}\Big((\partial_\sigma X^{\dot{r}}\gamma^{\dot{r}} - \mu X^{\dot{r}}\gamma^{\dot{r}}\Omega\Pi)(S^1_+ + \Omega S^2_+) + (\partial_\sigma X^{\hat{r}}\gamma^{\hat{r}} + \mu X^{\hat{r}'}\gamma^{\hat{r}'}\Omega\Pi)(S^1_- + \Omega S^2_-) \Big)_{\partial\Sigma} \tag{5.32}$$

and

$$q^-_{-\sigma}\Big|_{\partial\Sigma} = -\sqrt{\frac{1}{2p^+}}\Big((\partial_\sigma X^{\hat{r}}\gamma^{\hat{r}} - \mu X^{\hat{r}}\gamma^{\hat{r}}\Omega\Pi)(S^1_+ + \Omega S^2_+) + (\partial_\sigma X^{\dot{r}}\gamma^{\dot{r}} + \mu X^{\dot{r}'}\gamma^{\dot{r}'}\Omega\Pi)(S^1_- + \Omega S^2_-) \Big)_{\partial\Sigma} \,. \tag{5.33}$$

For $OD3$-branes, $q^-_{+\sigma}|_{\partial\Sigma}$ in eq. (5.32) identically vanishes since $\partial_\sigma X^{\hat{r}} = X^{\dot{r}} = X^{\hat{r}'} = 0$ and thus 4 dynamical supersymmetries q^-_+ are conserved as was shown in [9, 10]. In most cases except $OD3$-branes, $q^-_{\pm\sigma}|_{\partial\Sigma}$ appears not to vanish due to the presence of Neumann coordinates proportional to μ such as $X^{\dot{r}}$ and $X^{\hat{r}}$. However, there are some special cases satisfying

$$P_+\Omega = P_+\Pi \qquad \text{or} \qquad P_+\Pi\gamma \tag{5.34}$$

for some $OD5$-branes in eq. (2.17). For example, these are $OD5$-branes with $\Omega = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 \mp \gamma^5)\gamma^{34}$ and $\Omega = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 \mp \gamma^5)\gamma^{78}$ in table 1. The $OD5$-brane with $\Omega = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 \pm \gamma^5)\gamma^{37}$ in table 1 does not satisfy eq. (5.34).

For the $OD5$-branes satisfying eq. (5.34), $q^-_{\pm\sigma}|_{\partial\Sigma}$ vanish if the following boundary conditions are true, respectively:

$$\partial_\sigma X^{\dot{r}} - \mu X^{\dot{r}} = 0 \,, \qquad \forall \dot{r} \in PN \,, \tag{5.35}$$

$$\partial_\sigma X^{\hat{r}} = 0 \,, \qquad \forall \hat{r} \in ON \,, \tag{5.36}$$

$$X^{\hat{r}'} = 0 \,, \qquad \forall \hat{r}' \in OD \,, \tag{5.37}$$

and

$$\partial_\sigma X^{\hat{r}} - \mu X^{\hat{r}} = 0 \,, \qquad \forall \hat{r} \in ON \,, \tag{5.38}$$

$$\partial_\sigma X^{\dot{r}} = 0 \,, \qquad \forall \dot{r} \in PN \,, \tag{5.39}$$

$$X^{\dot{r}'} = 0 \,, \qquad \forall \dot{r}' \in PD \,. \tag{5.40}$$

Here abbreviations were used according to the index convention described in footnote 1. Note that some Neumann boundary conditions should be modified as in the D_+5-brane to preserve dynamical supersymmetry. Note also that the two kinds of dynamical supersymmetry, q^-_+ and q^-_-, cannot simultaneously be preserved since two sets of boundary condition, eqs. (5.35)–(5.37) and eqs. (5.38)–(5.40), cannot simultaneously be compatible with each other.

One may test whether an OD-brane with a constant flux as the D_-5-brane can preserve dynamical supersymmetry or not. The currents $q^-_{\pm\sigma}$ in eqs. (5.32) and (5.33) immediately show that it is not possible since the condition $\gamma^r = \pm\gamma^{r'}\Omega\Pi$ is never satisfied for OD-branes. For OD_-5-brane with $\Omega = -\gamma$, for example, $\gamma^{\hat{r}} \neq \pm\gamma^{\hat{r}'}\Omega\Pi$.

From the worldsheet point of view, the modification of the Neumann boundary condition in eq. (5.35) and eq. (5.38) corresponds to the addition of the following boundary terms, respectively:

$$S_B = \frac{1}{2\pi\alpha' p^+}\int d\tau \int_0^{2\pi\alpha'|p^+|} d\sigma F^{-\dot{r}}\partial_\sigma X^{\dot{r}} = \frac{\mu}{4\pi\alpha' p^+}\int_{\partial\Sigma} d\tau X^{\dot{r}}X^{\dot{r}}\,, \tag{5.41}$$

where the Born-Infeld flux $F^{-\dot{r}}$ is given by

$$F^{-\dot{r}} = \mu X^{\dot{r}}\,, \qquad \dot{r} = \dot{3}, \dot{4}\,, \tag{5.42}$$

and

$$S_B = \frac{1}{2\pi\alpha' p^+}\int d\tau \int_0^{2\pi\alpha'|p^+|} d\sigma F^{-\hat{r}}\partial_\sigma X^{\hat{r}} = \frac{\mu}{4\pi\alpha' p^+}\int_{\partial\Sigma} d\tau X^{\hat{r}}X^{\hat{r}}\,, \tag{5.43}$$

where

$$F^{-\hat{r}} = \mu X^{\hat{r}}\,, \qquad \hat{r} = \hat{1}, \hat{2}\,. \tag{5.44}$$

Thus we have to check whether the Born-Infeld fluxes such as (5.42) and (5.44) are consistent with the equation of motion for a worldvolume gauge field on the $OD5$-brane. We will show the details in appendix A that a worldvolume Chern-Simons coupling does give rise to the correct boundary conditions (5.35) and (5.38).

Let us take a more close look at the $OD5$-brane with $\Omega = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 \pm \gamma^5)\gamma^{37}$ as well as the $OD7$-brane with $\frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 \pm \gamma^5)\gamma^{3478}$ in table 1. If the boundary conditions in eqs. (5.36)–(5.37) hold, $q^-_{+\sigma}$ at the worldsheet boundary $\partial\Sigma$ is involved only with S^A_+ spinors whose all eigenvalues of the matrix $\Gamma = \Omega\Pi\Omega\Pi$ are 1. Thus we meet a similar situation to the case of $(+,-,n,n)$-brane. Indeed it turns out that some conserved dynamical supersymmetries exist in this case too. To see this, let us decompose the Neumann coordinates in the similar way as $\dot{r} = (\dot{r}_1, \dot{r}_2)$. For the $OD5$-brane, for example, $\dot{r}_1 = 3, \dot{r}_2 = 7$. For the projection matrix (5.21), we get

$$[P^{D+}_{\pm}, \gamma^{\dot{r}_1}] = 0\,, \qquad P^{D+}_{\pm}\gamma^{\dot{r}_2} = \gamma^{\dot{r}_2}P^{D+}_{\mp}\,. \tag{5.45}$$

Now define doubly projected supercharges

$$q^-_{+\pm} \equiv P^{D+}_{\pm}P_+(Q^{-1} - \Omega Q^{-2})\,. \tag{5.46}$$

Then, from eq. (5.32), we get $q^-_{+\pm\sigma}$ at the worldsheet boundary $\partial\Sigma$

$$\begin{aligned} q^-_{+\pm\sigma}\Big|_{\partial\Sigma} = -\sqrt{\frac{1}{2p^+}}\Big(&\partial_\sigma X^{\dot{r}_1}\gamma^{\dot{r}_1} \mp \mu X^{\dot{r}_1}\gamma^{\dot{r}_1})P^{D+}_{\pm}(S^1_+ + \Omega S^2_+) + \\ &+ (\partial_\sigma X^{\dot{r}_2}\gamma^{\dot{r}_2} \pm \mu X^{\dot{r}_2}\gamma^{\dot{r}_2})P^{D+}_{\mp}(S^1_+ + \Omega S^2_+)\Big)_{\partial\Sigma}. \end{aligned} \tag{5.47}$$

We see that two dynamical supersymmetries are preserved only if the boundary conditions eqs. (5.36)–(5.37) and the following Neumann boundary conditions given by

$$\left(\partial_\sigma X^{\dot{r}_1} \mp \mu X^{\dot{r}_1}\right)_{\partial\Sigma} = 0 = \left(\partial_\sigma X^{\dot{r}_2} \pm \mu X^{\dot{r}_2}\right)_{\partial\Sigma} \tag{5.48}$$

hold. The boundary condition (5.48) can also be realized by introducing a gauge field excitation of the similar form as eq. (5.27) with the Born-Infeld flux $F^{-\dot{r}}$ satisfying

$$\partial_{\dot{r}} F^{-\dot{r}} = 0\,, \qquad \dot{r} = (\dot{r}_1, \dot{r}_2)\,. \tag{5.49}$$

The results on the dynamical supersymmetry preserved by D-branes in table 1 have been summarized in table 2, in which we omitted BPS $D_\pm$-branes preserving 16 supersymmetries already identified in [20].

6. Supersymmetry of intersecting D-branes

We now analyze the supersymmetry of intersecting D-branes for which open string mode expansion has been given in section 4. In order to put our discussion on a general ground, we will not assume anything about the matrix Γ, so we are including the D-branes in (2.20) and (2.21) as well. The supersymmetry of intersecting $D_\pm$-branes was completely identified in [21] using the Green-Schwarz worldsheet formulation which can also be applied to more general class of D-branes under consideration. In what follows, all supercharges are assumed to be expressed in view of the D-brane at $\sigma = 0$ in the same way as the mode expansion in section 4.

In general, the unbroken supersymmetry of intersecting D-branes is the 'intersection' of supersymmetries preserved by each brane. The intersection is characterized by the projection matrices $I_\pm$ in eq. (4.4). In section 5, we showed that the conserved supersymmetry of a single D-brane is described by two kinds of projection matrices, $P_\pm$ and $P_\pm^{D+}$, in eqs. (3.7) and (5.21). Thus the unbroken supersymmetry of intersecting D-branes shall be completely characterized by these three kinds of projection matrices. From the definitions of the projection matrices, one can see that they are mutually commuting, viz.,

$$[I_\pm, P_\pm] = [I_\pm, P_\pm^{D+}] = [P_\pm, P_\pm^{D+}] = 0\,, \tag{6.1}$$

if the condition (4.10) holds. From now on, we will assume it.

For an intersection of half BPS D-branes, it was shown in [21] that the supersymmetry of intersecting D-branes is given by $I_\pm q^+$ and $I_\pm q^-$ where the kinematical supercharge q^+ is defined by eq. (5.5) and (5.10) and the dynamical supercharge q^- is by eq. (5.15). For the present problem, however, the supersymmetry of a single D-brane is in general a subset of q^+ and q^-, represented by $P_\pm$ and $P_\pm^{D+}$, as illustrated in table 2. Nevertheless, since all the projection matrices mutually commute as in eq. (6.1), we first find the condition for the supercharges $I_\pm q^+$ and $I_\pm q^-$ to be conserved and then construct unbroken supersymmetries using the projection matrices $P_\pm$ and $P_\pm^{D+}$. This immediately implies that intersecting D-branes preserve no supersymmetry when $\Gamma_0^2 = -1$ since q^+ and q^- were originally not conserved quantities.

JHEP03(2004)058

Following the recipe explained above, it is simple to check from the conservation laws (5.7) and (5.12) using the identities (4.9), (4.12), and (4.13) that the supercharge

$$q^+ = I_\pm(q^+_{D_-} + q^+_{D_+}) \tag{6.2}$$

is conserved for A-type branes with I_+ and for B-type branes with I_- only if $\Gamma_0^2 = 1$. Here $q^+_{D_-}$ and $q^+_{D_+}$ are defined by eq. (5.9) and eq. (5.14), respectively. Since we only required the condition (4.10) for the matrices Γ_0 and Γ_π, there are two kinds of intersection to preserve the kinematical supersymmetry:

$$\Gamma_0 = \Gamma_\pi\,, \tag{6.3}$$

$$\Gamma_0 = \Gamma_\pi \gamma\,. \tag{6.4}$$

The special case, $\Gamma_0 = \pm 1 = \Gamma_\pi$, in eq. (6.3) was already analyzed in [21] and their unbroken supersymmetry was completely identified. eq. (6.4) implies that there are supersymmetric intersections between $D_\pm$-branes and $OD5_\pm$-branes as well as between ODp-branes with, e.g., $\Gamma_0 = \pm\gamma^{1256}$ and ODq-branes with $\Gamma_\pi = \pm\gamma^{3478}$.

For a given class in (6.3) or (6.4), the number of unbroken kinematical supersymmetry depends on the total number of ND and DN directions which is captured by the matrix in (4.16). Since a single D-brane originally had $8 = 4_{D_-} + 4_{D_+}$ kinematical supersymmetries as shown in table 2, intersecting D-branes, depending on the total number of ND directions, will have $8 = 4_{D_-} + 4_{D_+}$, $4 = 2_{D_-} + 2_{D_+}$, or 0 kinematical supersymmetries according to eq. (6.2).

Now we will discuss the dynamical supersymmetry of intersecting D-branes. It is useful to recall the (anti-)commutation relations between $\gamma^I = \{\gamma^r, \gamma^{r'}, \gamma^i, \gamma^{i'}\}$, Ω_0 and Ω_π to find conserved dynamical supersymmetries:

$$\{\gamma^r, \Omega_0\} = \{\gamma^i, \Omega_0\} = [\gamma^{r'}, \Omega_0] = [\gamma^{i'}, \Omega_0] = 0\,, \tag{6.5}$$

$$\{\gamma^r, \Omega_\pi\} = \{\gamma^{i'}, \Omega_\pi\} = [\gamma^{r'}, \Omega_\pi] = [\gamma^i, \Omega_\pi] = 0\,. \tag{6.6}$$

Here we adopted the indices explained in footnote 3.

To follow the same strategy explained above, we introduce a dynamical supercharge of intersecting D-branes defined by[6]

$$q^- = I_\pm(Q^{-1} - \Omega Q^{-2}) \tag{6.7}$$

where I_+ is for A-type branes and I_- for B-type branes. Since the supercharge in eq. (6.7) is just the projection of that in eq. (5.15), it is obvious that the supercharge in eq. (6.7) also satisfies the conservation law (5.16). Using the (anti-)commutation relations (6.5) and (6.6), it is easy to find q^-_σ from eq. (5.17) which is given by

$$q^-_\sigma = \sqrt{\frac{1}{2p^+}}\Big((\partial_\tau X^r\gamma^r - \partial_\sigma X^{r'}\gamma^{r'})I_\pm(S^1 - \Omega_0 S^2) + (\partial_\tau X^i\gamma^i - \partial_\sigma X^{i'}\gamma^{i'})I_\mp(S^1 - \Omega_0 S^2) +$$

[6] Note that there is a minor difference between eq. (6.7) and that in [21] due to the different definition about q^- mentioned in footnote 4.

$$
\begin{aligned}
&+(\partial_\tau X^{r'}\gamma^{r'} - \partial_\sigma X^r\gamma^r) I_\pm (S^1 + \Omega_0 S^2) + (\partial_\tau X^{i'}\gamma^{i'} - \partial_\sigma X^i\gamma^i) I_\mp (S^1 + \Omega_0 S^2) + \\
&+ \mu X^r \gamma^r \Omega_0 \Pi I_\pm (S^1 + \Gamma_0\Omega_0 S^2) + \mu X^i\gamma^i\Omega_0\Pi I_\mp (S^1 + \Gamma_0\Omega_0 S^2) - \\
&- \mu X^{r'}\gamma^{r'}\Omega_0\Pi I_\pm (S^1 - \Gamma_0\Omega_0 S^2) - \mu X^{i'}\gamma^{i'}\Omega_0\Pi I_\mp (S^1 - \Gamma_0\Omega_0 S^2)\Big) .
\end{aligned}
\tag{6.8}
$$

Since the case $\Gamma_0^2 = -1$ cannot preserve any supersymmetry, we will focus on the case $\Gamma_0^2 = 1$. To find the condition for $q_\sigma^-|_{\partial\Sigma}$ to vanish, we first decompose the supercharge q^- in eq. (6.7) in terms of the projection matrix $P_\pm = \frac{1}{2}(1 \pm \Gamma_0)$:

$$
q_\pm^- \equiv P_\pm I_\pm (Q^{-1} - \Omega Q^{-2}) . \tag{6.9}
$$

Using eqs. (5.30)–(5.31) and adopting the notation in footnote 3, it is then easy to see that

$$
\begin{aligned}
q_{+\sigma}^-|_{\partial\Sigma} = -\sqrt{\frac{1}{2p^+}}\Big(&(\partial_\sigma X^{\dot r}\gamma^{\dot r} - \mu X^{\dot r}\gamma^{\dot r}\Omega_0\Pi) I_\pm (S_+^1 + \Omega_0 S_+^2) + \\
&+ (\partial_\sigma X^{\hat r}\gamma^{\hat r} + \mu X^{\hat r'}\gamma^{\hat r'}\Omega_0\Pi) I_\pm (S_-^1 + \Omega_0 S_-^2) + \\
&+ (\partial_\sigma X^{\dot i}\gamma^{\dot i} - \mu X^{\dot i}\gamma^{\dot i}\Omega_0\Pi) I_\mp (S_+^1 + \Omega_0 S_+^2) + \\
&+ (\partial_\sigma X^{\hat i}\gamma^{\hat i} + \mu X^{\hat i'}\gamma^{\hat i'}\Omega_0\Pi) I_\mp (S_-^1 + \Omega_0 S_-^2) + \\
&+ (\partial_\sigma X^{\dot i'}\gamma^{\dot i'} + \mu X^{\dot i'}\gamma^{\dot i'}\Omega_0\Pi) I_\mp (S_+^1 - \Omega_0 S_+^2) + \\
&+ (\partial_\sigma X^{\hat i'}\gamma^{\hat i'} - \mu X^{\hat i}\gamma^{\hat i}\Omega_0\Pi) I_\mp (S_-^1 - \Omega_0 S_-^2)\Big)
\end{aligned}
\tag{6.10}
$$

and

$$
\begin{aligned}
q_{-\sigma}^-|_{\partial\Sigma} = -\sqrt{\frac{1}{2p^+}}\Big(&(\partial_\sigma X^{\dot r}\gamma^{\dot r} + \mu X^{\dot r'}\gamma^{\dot r'}\Omega_0\Pi) I_\pm (S_-^1 + \Omega_0 S_-^2) + \\
&+ (\partial_\sigma X^{\hat r}\gamma^{\hat r} - \mu X^{\hat r}\gamma^{\hat r}\Omega_0\Pi) I_\pm (S_+^1 + \Omega_0 S_+^2) + \\
&+ (\partial_\sigma X^{\dot i'}\gamma^{\dot i'} - \mu X^{\dot i}\gamma^{\dot i}\Omega_0\Pi) I_\mp (S_-^1 - \Omega_0 S_-^2) + \\
&+ (\partial_\sigma X^{\hat i'}\gamma^{\hat i'} + \mu X^{\hat i'}\gamma^{\hat i'}\Omega_0\Pi) I_\mp (S_+^1 - \Omega_0 S_+^2) \\
&+ (\partial_\sigma X^{\dot i}\gamma^{\dot i} + \mu X^{\dot i'}\gamma^{\dot i'}\Omega_0\Pi) I_\mp (S_-^1 + \Omega_0 S_-^2) + \\
&+ (\partial_\sigma X^{\hat i}\gamma^{\hat i} - \mu X^{\hat i}\gamma^{\hat i}\Omega_0\Pi) I_\mp (S_+^1 + \Omega_0 S_+^2)\Big)
\end{aligned}
\tag{6.11}
$$

where we dropped obviously vanishing terms at the boundary $\partial\Sigma$. The second identities in eqs. (4.12)–(4.13) played a crucial role to derive the above result.

It is straightforward to find intersecting D-brane configurations preserving some dynamical supersymmetry from eqs. (6.10)–(6.11). Here the second identities in eqs. (4.12)–(4.13) will play a crucial role again to do this job. Also recall the following facts

$$
\gamma Q^{-A} = -Q^{-A} , \qquad \gamma S^A = S^A . \tag{6.12}
$$

- $\Gamma_0 = \Gamma_\pi = -1$: This case is exactly the $D_- - D_-$ brane intersections studied in [21]. The worldsheet current is given by eq. (6.11) where only $S_-^A = S^A$ spinors survive. From eq. (6.8), one can see that there can be supersymmetric intersections of D_-5-branes with a constant flux when $\sharp_{ND} = 0, 4, 8$. For example, let us consider a $\sharp_{ND} = 4$ intersection of $D5$-brane extended along $(1, 2, 3, 5)$ directions with flux

$F_{+5} = \mu q_1$ and $D5$-brane along $(1,2,4,6)$ directions with flux $F_{+6} = \mu q_2$. In this case, we have $\Omega_0\Pi = -\gamma^{45}$, $\Omega_\pi\Pi = \gamma^{36}$, and $\Omega_0^T\Omega_\pi = \gamma^{3456}$. It is easy to check that $q_\sigma^-|_{\partial\Sigma} = 0$ if the following boundary conditions are satisfied

$$\begin{aligned}\partial_\sigma X^{1,2,3}|_{\sigma=0} &= 0 = X^{6,7,8}|_{\sigma=0}, \qquad (\partial_\sigma X^5 - \mu q_1)_{\sigma=0} = 0 = (X^4 - q_1)_{\sigma=0}\,,\\ \partial_\sigma X^{1,2,4}|_{\sigma=\pi} &= 0 = X^{5,7,8}|_{\sigma=\pi}, \qquad (\partial_\sigma X^6 - \mu q_2)_{\sigma=\pi} = 0 = (X^3 - q_2)_{\sigma=\pi}\end{aligned} \tag{6.13}$$

Similarly, for a $\sharp_{ND} = 8$ intersection of $D5$-brane extended along $(1,2,3,5)$ directions with flux $F_{+5} = \mu q_1$ and $D5$-brane along $(4,6,7,8)$ directions with flux $F_{+4} = \mu q_2$, one can also check that $q_\sigma^-|_{\partial\Sigma} = 0$.

- $\Gamma_0 = \Gamma_\pi = 1$: This case corresponds to the $D_+ - D_+$ brane intersections. The worldsheet current is given by eq. (6.10) where only $S_+^A = S^A$ spinors survive. It is convenient to decompose the supercharge q_+^- in terms of the projection matrix $P_\pm^{D+} = \frac{1}{2}(1 \pm \Omega_0\Pi)$ as in eq. (5.22). Note that the third term in eq. (6.10) gives a constraint only at $\sigma = 0$, i.e., a Neumann boundary condition while the fifth term does only at $\sigma = \pi\alpha$, also a Neumann boundary condition. Using eqs. (5.23)–(5.24), we get

$$(\partial_\sigma X^{r_1} \mp \mu X^{r_1})_{\partial\Sigma} = 0\,, \qquad (\partial_\sigma X^{r_2} \pm \mu X^{r_2})_{\partial\Sigma} = 0\,, \tag{6.14}$$

$$(\partial_\sigma X^{i_1} \mp \mu X^{i_1})_{\sigma=0} = 0\,, \qquad (\partial_\sigma X^{i_2} \pm \mu X^{i_2})_{\sigma=0} = 0\,, \tag{6.15}$$

$$(\partial_\sigma X^{i'_1} \mp \mu X^{i'_1})_{\sigma=\pi\alpha} = 0\,, \qquad (\partial_\sigma X^{i'_2} \pm \mu X^{i'_2})_{\sigma=\pi\alpha} = 0\,, \tag{6.16}$$

where the upper sign is for P_+^{D+} and the lower one for P_-^{D+}. We see that the $D_+ - D_+$ brane intersections can preserve some dynamical supersymmetries as long as the worldvolume fluxes are consistently aligned as the way in eqs. (6.14)–(6.16). Consider, for example, $(+,-,4,0) - (+,-,1,1)$ brane intersection. The supersymmetric boundary conditions must be

$$\partial_\sigma X^{r_1} - \mu X^{r_1} = 0\,, \qquad \partial_\sigma X^{i_1} - \mu X^{i_1} = 0\,, \qquad \partial_\sigma X^{i'_2} + \mu X^{i'_2} = 0 \tag{6.17}$$

for $r_1 = 1$, $i_1 = 2,3,4$ and $i'_2 = 5$, for example. However, $(+,-,4,0) - (+,-,0,4)$ brane intersection cannot satisfy eq. (6.15) and eq. (6.16) simultaneously and thus preserves no dynamical supersymmetry as was shown in [21].

- $\Gamma_0 = \Gamma_\pi = -\gamma$: This case is the $OD_-5 - OD_-5$ brane intersection. The worldsheet current is given by eq. (6.10) where only $S_-^A = S^A$ spinors survive since eq. (6.12). The dynamical supersymmetry can be preserved as long as the two branes are placed at origin, viz.,

$$X^{\hat{r}'}|_{\partial\Sigma} = X^{\hat{i}'}|_{\sigma=0} = X^{\hat{i}}|_{\sigma=\pi\alpha} = 0\,, \tag{6.18}$$

and they satisfy the usual Neumann boundary conditions. These boundary conditions are the same as the $D_-5 - D_-5$ brane case as expected.

- $\Gamma_0 = \Gamma_\pi = \gamma$: This case is the $OD_+5 - OD_+5$ brane intersection. The worldsheet current is given by eq. (6.11) where only $S_+^A = S^A$ spinors survive since eq. (6.12). Any dynamical supersymmetry is not preserved as expected.

- $\Gamma_0 = \Gamma_\pi = \pm\gamma^{1256}$: First, note that for $\Omega_0 = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 \mp \gamma^5)\gamma^{34}$ in table 1

$$\Omega_0 \Pi = P_+ + \gamma^{16} P_- \tag{6.19}$$

and for $\Omega_0 = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 \mp \gamma^5)\gamma^{78}$

$$\Omega_0 \Pi = \mp(P_+ - \gamma^{16} P_-)\gamma\,. \tag{6.20}$$

For the first case (6.19), there are two ways to preserve dynamical supersymmetries. One set can be read off from eq. (6.10):

$$\begin{aligned} X^{\hat{r}'}|_{\partial\Sigma} &= X^{\hat{i}'}|_{\sigma=0} = X^{\hat{i}}|_{\sigma=\pi\alpha} = 0\,, \\ \partial_\sigma X^{\hat{r}}|_{\partial\Sigma} &= \partial_\sigma X^{\hat{i}}|_{\sigma=0} = \partial_\sigma X^{\hat{i}'}|_{\sigma=\pi\alpha} = 0\,, \\ (\partial_\sigma X^{\dot{r}} - \mu X^{\dot{r}})_{\partial\Sigma} &= (\partial_\sigma X^{\dot{i}} - \mu X^{\dot{i}})_{\sigma=0} = (\partial_\sigma X^{\dot{i}'} + \mu X^{\dot{i}'})_{\sigma=\pi\alpha} = 0\,. \end{aligned} \tag{6.21}$$

The other set can be done from eq. (6.11):

$$\begin{aligned} X^{\dot{r}'}|_{\partial\Sigma} &= X^{\dot{i}'}|_{\sigma=0} = X^{\dot{i}}|_{\sigma=\pi\alpha} = 0\,, \\ \partial_\sigma X^{\dot{r}}|_{\partial\Sigma} &= \partial_\sigma X^{\dot{i}}|_{\sigma=0} = \partial_\sigma X^{\dot{i}'}|_{\sigma=\pi\alpha} = 0\,, \\ (\partial_\sigma X^{\hat{r}} - \mu X^{\hat{r}})_{\partial\Sigma} &= (\partial_\sigma X^{\hat{i}} - \mu X^{\hat{i}})_{\sigma=0} = (\partial_\sigma X^{\hat{i}'} + \mu X^{\hat{i}'})_{\sigma=\pi\alpha} = 0\,. \end{aligned} \tag{6.22}$$

For the second case (6.20), the result is similar.

For the $OD5$-brane with $\Omega_0 = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 \pm \gamma^5)\gamma^{37}$ and the $OD7$-brane with $\Omega_0 = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 \pm \gamma^5)\gamma^{3478}$ in table 1, we can apply the same method as section 5. After defining (now triply) projected supercharges like as (5.46), one can easily find the supersymmetric boundary conditions. These are the same as eqs. (6.21)–(6.22) except that the Neumann boundary conditions modified by gauge field condensates are split into two SO(4) directions in a way consistent with the equation of motion such as eq. (5.49).

There is a subtle point when we consider an intersection of the $OD5$-brane with $\Omega_0 = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 \pm \gamma^5)\gamma^{34}$ and the $OD5$-brane with $\Omega_\pi = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 \pm \gamma^5)\gamma^{37}$ as a typical example. If we choose the former brane as a reference brane, the projection matrix (5.21) is no longer needed to construct a conserved supercharge. However, if we choose the latter one, the projection matrix, at first sight, seems to be necessary to construct a conserved supercharge. This seems to introduce a contradictory result that the conserved dynamical supersymmetry depends on our choice of a reference brane. This puzzle can be resolved as follows. Suppose that we choose the $OD5$-brane described by Ω_π as a reference brane. Since we have already had the projection matrix $I_\pm$, the gluing matrix Ω_π can be represented by Ω_0 as follows

$$\Omega_\pi = \Omega_0(I_+ - I_-)\,. \tag{6.23}$$

Thus it is not necessary to further introduce the projection matrices (5.21) and the dynamical supersymmetry is not further reduced in the case at hand. The other cases, e.g. the $OD7$-branes with $\Omega_\pi = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 \pm \gamma^5)\gamma^{3478}$, are similar too.

D-brane type	Intersection	q^+	q^-
$\Gamma_0 = \Gamma_\pi = -1$	$D_-p - D_-q$	ν	$\frac{1}{2}\,\mathrm{Tr}(1-\gamma)I_\pm$
	$\mathbf{D}_-5 - \mathbf{D}_-5$	ν	$\frac{1}{2}\,\mathrm{Tr}(1-\gamma)I_\pm$
$\Gamma_0 = \Gamma_\pi = 1$	$D_+1 - \mathbf{D}_+p$	ν	$\frac{1}{2}\,\mathrm{Tr}(1-\gamma)P_\pm^{D_+}I_\pm$
	$\mathbf{D}_+p - \mathbf{D}_+q$	ν	$\frac{1}{2}\,\mathrm{Tr}(1-\gamma)P_\pm^{D_+}I_\pm$
$\Gamma_0 = \Gamma_\pi = -\gamma$	$OD_-5 - OD_-5$	ν	$\frac{1}{2}\,\mathrm{Tr}(1-\gamma)I_\pm$
$\Gamma_0 = \Gamma_\pi = \gamma$	$OD_+5 - OD_+5$	ν	0
$\Gamma_0 = \Gamma_\pi = \pm\gamma^{1256}$	$\mathbf{OD}p - \mathbf{OD}q$	ν	$\frac{1}{2}\,\mathrm{Tr}(1-\gamma)P_\pm^{D_+}P_\pm I_\pm$
$\Gamma_0 = -1, \Gamma_\pi = -\gamma$	$D_-p - OD_-5$	ν	0
$\Gamma_0 = 1, \Gamma_\pi = \gamma$	$D_+p - OD_+5$	ν	0
$\Gamma_0 = \pm\gamma^{1256}, \Gamma_\pi = \pm\gamma^{3478}$	$ODp - ODq$	ν	0

Table 3: Supersymmetry of intersecting D-branes. $\nu = n_{D_-} + n_{D_+}$ is the number of unbroken kinematical supersymmetry where $n_{D_-} = \frac{1}{2}\,\mathrm{Tr}(1+\gamma)P_-I_\pm$ (D_--type) and $n_{D_+} = \frac{1}{2}\,\mathrm{Tr}(1+\gamma)P_+I_\pm$ (D_+-type). A D-brane with a gauge field condensate is denoted by the boldface.

- $\Gamma_0 = -1$, $\Gamma_\pi = -\gamma$: This case is the $D_-p - OD_-5$ brane intersection. However any dynamical supersymmetry cannot be preserved. The reason is simple: If we choose $P_\pm = \frac{1}{2}(1 \pm \Gamma_0)$, the non-vanishing supercharge is q_-^-. As it should be, the physics must be equally well described in view of the D-brane at $\sigma = \pi\alpha$ as well where $P_\pm = \frac{1}{2}(1 \pm \Gamma_\pi)$. Then the non-vanishing supercharge is now q_+^-. But these two charges cannot simultaneously be compatible with each other as we discussed below eq. (5.40). This kind of thing does not happen for kinematical supersymmetry since $\gamma Q^{+A} = Q^{+A}$, which is a reason why it can be preserved in the case of $\Gamma_0 = \Gamma_\pi\gamma$.

- $\Gamma_0 = 1$, $\Gamma_\pi = \gamma$: This case is the $D_+p - OD_+5$ brane intersection. Similarly this case has no dynamical supersymmetry.

- $\Gamma_0 = \pm\gamma^{1256}$, $\Gamma_\pi = \pm\gamma^{3478}$: This case corresponds to the $ODp - ODq$ brane intersection. This case has no dynamical supersymmetry either since $(1 \pm \gamma^{1256})(1 \pm \gamma^{3478}) = (1 \pm \gamma^{1256})(1+\gamma)$.

We summarized the supersymmetry preserved by various configurations of intersecting D-branes in table 3. The number of each type of kinematical supersymmetries depends on the total number of ND and DN directions in a way determined by the projection matrix $I_\pm P_-$ (D_--type) or $I_\pm P_+$ (D_+-type) as was shown in eq. (6.2). The number of dynamical supersymmetries listed in table 3 may be counted using the relations (6.19), (6.20), and (6.23).

7. Supersymmetric curved D-branes

It was shown in [9, 10] that the plane wave background (1.1) admits supersymmetric curved D-branes as well as oblique D-branes. We will also classify the supersymmeric curved D-branes including those not mentioned in the previous literatures.

JHEP03(2004)058

To discuss curved D-branes, it is convenient to introduce the complex coordinates as

$$Z^i = X^i + iX^{i+4}\,, \qquad i = 1, \ldots, 4 \tag{7.1}$$

and the fermionic creation and annihilation operators

$$b_i = \frac{1}{2}(\gamma^i - i\gamma^{i+4})\,, \qquad b_i^\dagger = \frac{1}{2}(\gamma^i + i\gamma^{i+4})\,, \tag{7.2}$$

$$\{b_i, b_j^\dagger\} = \delta_{ij}\,, \qquad i,j = 1, \ldots, 4\,. \tag{7.3}$$

Using a general $\mathcal{N} = (2,2)$ supersymmetric worldsheet theory, Hikida and Yamaguchi showed in [9] that a supersymmetric curved D-brane is wrapped on a complex submanifold and the superpotential due to the geometry (1.1) should be constant on its worldvolume. The maximally supersymmetric plane wave (1.1) has a special superpotential

$$W = i(Z_1^2 + Z_2^2 + Z_3^3 + Z_4^2)\,. \tag{7.4}$$

Thus possible supersymmetric curved D-branes in the background (1.1) can be exhausted as follows:

$$D7: \;\; Z_1^2 + Z_2^2 + Z_3^3 + Z_4^2 = c_1\,, \tag{7.5}$$

$$D5: \;\; Z_1^2 + Z_2^2 + Z_3^3 = c_1\,, \qquad Z_4 = c_2\,, \tag{7.6}$$

$$Z_1^2 + Z_2^2 = c_1\,, \qquad Z_3^2 + Z_4^2 = c_2\,, \tag{7.7}$$

$$Z_1^2 + Z_2^2 = c_1\,, \qquad Z_3 = iZ_4\,, \tag{7.8}$$

$$D3: \;\; Z_1^2 + Z_2^2 = c_1\,, \qquad Z_3 = c_2\,, \qquad Z_4 = c_3\,, \tag{7.9}$$

where c_i's are complex constants. Following the ref. [10], we identify the gluing matrices Ω for the above curved D-branes as follows:

$$D7: \;\; \Omega = -i[b_i, b_j^\dagger]\frac{\bar{Z}^i Z^j}{\sum_{k=1}^4 |Z_k|^2}\,, \qquad i,j = 1, \ldots, 4\,, \tag{7.10}$$

$$D5: \;\; \Omega = -i[b_i, b_j^\dagger]\gamma^{48}\frac{\bar{Z}^i Z^j}{\sum_{k=1}^3 |Z_k|^2}\,, \qquad i,j = 1,2,3\,, \tag{7.11}$$

$$\Omega = -[b_i, b_j^\dagger][b_k, b_l^\dagger]\frac{\bar{Z}^i Z^j}{|Z_1|^2 + |Z_2|^2}\frac{\bar{Z}^k Z^l}{|Z_3|^2 + |Z_4|^2}\,, \qquad i,j = 1,2\,, \qquad k,l = 3,4\,, \tag{7.12}$$

$$\Omega = -\frac{i}{2}[b_i, b_j^\dagger](\gamma^3 + \gamma^8)(\gamma^4 - \gamma^7)\frac{\bar{Z}^i Z^j}{|Z_1|^2 + |Z_2|^2}\,, \qquad i,j = 1,2\,, \tag{7.13}$$

$$D3: \;\; \Omega = -i[b_i, b_j^\dagger]\gamma^{3478}\frac{\bar{Z}^i Z^j}{|Z_1|^2 + |Z_2|^2}\,, \qquad i,j = 1,2\,, \tag{7.14}$$

where Ω's are now taken to be the product of the γ-matrices associated to the Dirichlet directions. The $D7$-brane in eq. (7.5) was previously discussed in [9, 10] and the $D5$-brane in eq. (7.6) was in [10]. Note that $\Omega^T\Omega = 1$ always and $\Omega^2 = -1$ for $D7$- and $D3$-branes while $\Omega^2 = 1$ for $D5$-branes.

The fermionic boundary condition is still given by eq. (2.10), but with the gluing matrix Ω depending on the nontrivial worldvolume geometry of D-branes as shown in eqs. (7.10)–(7.14). Furthermore, it turned out [10] that an open string on curved D-branes satisfies very complicated Neumann boundary conditions modified by fermion bilinears. Thus it is in practice difficult to find the open string mode expansion for the curved D-branes and to generalize the worldsheet formulation for the supersymmetry analysis in section 5 to the present case. Nevertheless it is possible to identify unbroken supersymmetries of the curved D-branes described by eqs. (7.10)–(7.14) by applying the results given in [25, 9, 10].

In order to discuss the supersymmetry, it is convenient to introduce a Fock space notation. The vacuum is defined to be the spinor annihilated by all b_i's and denoted by $|\downarrow\rangle = (-,-,-,-)$. And the top spinor annihilated by all $b_i^\dagger$'s is denoted by $|\uparrow\rangle = (+,+,+,+)$. The other fourteen spinors are defined as $|i\downarrow\rangle = b_i^\dagger|\downarrow\rangle = (+,-,-,-)_{i=1}$, $|i\uparrow\rangle = b_i|\uparrow\rangle = (-,+,+,+)_{i=1}$, and $|ij\rangle = b_i^\dagger b_j^\dagger|\downarrow\rangle = (+,+,-,-)_{i,j=1,2}$ or $b_i b_j|\uparrow\rangle = (-,-,+,+)_{i,j=1,2}$. The IIB chiral spinors $\epsilon_\pm$ in light-cone are 8-component complex spinors. ϵ_+ is related to the supersymmetries that are non-linearly realized on the worldsheet, i.e., the kinematical supersymmetries and ϵ_- is to those that are linearly realized on the worldsheet, namely, the dynamical supersymmetries.[7] It was shown in [25] that the supersymmetric solutions $\epsilon_\pm$ for the maximally supersymmetric background (1.1) can be parameterized in this Fock space as

$$\epsilon_+ = \kappa_1 + i\kappa_2 = \beta^i \bar{Z}^i|i\downarrow\rangle + \delta^i Z^i|i\uparrow\rangle\,, \tag{7.15}$$

$$\epsilon_- = \epsilon_1 + i\epsilon_2 = \alpha|\downarrow\rangle + \eta^{ij}|ij\rangle + \zeta|\uparrow\rangle\,, \tag{7.16}$$

where $\alpha, \eta^{ij}, \zeta, \beta^i$ and δ^i are complex constants.

For an open string on a curved D-brane described by a gluing matrix Ω, we have the boundary conditions [10]

$$\kappa_1 = \Omega\kappa_2\,, \tag{7.17}$$

$$\epsilon_1 = -\Omega\epsilon_2\,. \tag{7.18}$$

In terms of the Fock basis (7.15) and (7.16), the boundary conditions (7.17) and (7.18), respectively, can be expressed as

$$\Omega|i\downarrow\rangle = a_i|i\downarrow\rangle\,, \qquad \Omega|i\uparrow\rangle = b_i|i\uparrow\rangle\,, \tag{7.19}$$

$$\Omega|\downarrow\rangle = \Omega|ij\rangle = \Omega|\uparrow\rangle = \text{constant}\,, \tag{7.20}$$

where a_i and b_i are complex constants and no summation for i is assumed in eq. (7.19). We can explicitly solve eqs. (7.19)–(7.20) for each Ω in (7.10)–(7.14) which allows us to identify the unbroken supersymmetries. We summarized the possible solutions corresponding to kinematical and dynamical supersymmetries in table 4. The spinors $|4\uparrow\rangle, |4\downarrow\rangle$ for eq. (7.11) in table 4 satisfy $\Pi\Omega\Pi\Omega = 1$, which thus give rise to the D_+-brane kinematical supersymmetry while the other two do $\Pi\Omega\Pi\Omega = -1$ giving rise to the D_--brane kinematical supersymmetry. The kinematical supersymmetry for eq. (7.11) is not inconsistent with [10] since they concerned only the D_--brane kinematical supersymmetry satisfying $\Pi\Omega\Pi\Omega = -1$.

[7]Note that we are using the different convention from that in [25, 9, 10] according to our assignment of SO(8) chirality for the kinematical and the dynamical supersymmetries. To be with this convention, we take the vacuum $|\downarrow\rangle$ as a negative chirality spinor, viz., $\gamma|\downarrow\rangle = -|\downarrow\rangle$.

D-brane type	W	Ω	q^+	q^-
$D7$	(7.5)	(7.10)	0	$\vert\downarrow\rangle, \vert\uparrow\rangle$
$D5$	(7.6)	(7.11)	$\vert 4\uparrow\rangle, \vert 4\downarrow\rangle$	$\vert\downarrow\rangle, \vert\uparrow\rangle$
	(7.7)	(7.12)	0	$\vert\downarrow\rangle, \vert 12\rangle, \vert 34\rangle, \vert\uparrow\rangle$
	(7.8)	(7.13)	$\vert 4\uparrow\rangle, \vert 3\uparrow\rangle, \vert 3\downarrow\rangle, \vert 4\downarrow\rangle$	$\vert\downarrow\rangle, \vert 12\rangle, \vert 34\rangle, \vert\uparrow\rangle$
$D3$	(7.9)	(7.14)	$\vert 4\uparrow\rangle, \vert 3\uparrow\rangle, \vert 3\downarrow\rangle, \vert 4\downarrow\rangle$	$\vert\downarrow\rangle, \vert 12\rangle, \vert 34\rangle, \vert\uparrow\rangle$

Table 4: Supersymmetric curved D-branes. We indicated the equation number for the superpotential W and the gluing matrix Ω.

There may be curved D-branes preserving only kinematical supersymmetries on which W is not constant and thus dynamical supersymmetries are completely broken. We will not explore this kind of curved D-branes.

8. Discussion

The aim of this paper was to give a systematic classification of D-branes in the type-IIB plane wave background. Of course, our work does not mean to complete this goal even in the maximally supersymmetric plane wave background (1.1). We only considered a free string theory in the plane wave background and static D-branes. Moreover we used the light-cone open string theory where only longitudinal D-branes are visible. String interactions may break certain symmetries. Furthermore there can exist less supersymmetric solutions than those found in this paper, e.g., by considering D-branes intersecting at general angles [26, 27] or by considering more general plane wave backgrounds [25].

Let us discuss some related issues mentioned above. In this paper we considered only static D-branes. However one can generate new D-branes, symmetry related D-branes which are in general time-dependent branes [17], from static D-branes using the symmetries being in the action (2.1) and the target spacetime (1.1) but broken by D-branes, e.g., the translation and the boost generators along the transverse directions, $P^{r'}$ and $J^{+r'}$. A rotating D-brane and a giant graviton in Penrose limit can be described by these symmetry related boundary conditions which preserve the same amount of supersymmetry [17].

In this paper we studied parallel and orthogonally intersecting D-branes only. It will be straightforward to extend our analysis to D-branes intersecting at general angles [21]. Since the rotational symmetry is reduced to $\mathrm{SO}(4) \times \mathrm{SO}(4)'$, there are only two kinds of supersymmetric intersection at general angles, resulting in less supersymmetric D-brane configurations. One is generated by $\mathrm{SU}(2) \subset \mathrm{SO}(4)$ or $\mathrm{SO}(4)'$ and the other is generated by $\mathrm{SU}(2) \times \mathrm{SU}(2) \subset \mathrm{SO}(4) \times \mathrm{SO}(4)'$. The former case preserves the supersymmetry by half after rotation while the latter does by quarter like in the flat spacetime.

The dynamical supersymmetry of an OD-brane also appears to be broken when the brane is located away from the origin of transverse space and the breaking terms depend only on the boundary value of some Dirichlet coordinates like as D_--branes. It can be shown [17], however, that the broken dynamical supersymmetries can be restored by modifying transformation rules using a worldsheet symmetry realized in the action (2.1).

JHEP03(2004)058

However it was argued in [28] that these extra symmetries in the case of D-branes are not respected by string interactions. The same thing happens for the kinematical supersymmetry preserved by D_+-branes. Thus this result implies that the dynamical supersymmetry of a D_--brane or a related OD-brane located away from the origin and the kinematical supersymmetry of a D_+-brane or a related OD-brane are broken down by turning on the string interaction.

The isometry in the plane wave background (1.1) is indeed $\mathrm{SO}(4) \times \mathrm{SO}(4)' \times \mathbb{Z}_2$ where the first SO(4) is a remnant of the SO(4,2) isometry group of AdS_5 and the second SO(4)′ is a remnant of the SO(6) isometry group of S^5. The peculiar $\mathbb{Z}_2$ symmetry exchanges these two SO(4)'s as defined by (2.12). As we speculated in footnote 2, the $\mathbb{Z}_2$ symmetry seems to play an important role in the existence of OD-branes. This discrete symmetry survives only in the strict plane wave limit. This symmetry is broken if we perturb slightly away from the limit to $AdS_5 \times S^5$. In the pp-wave/SYM duality, the rotation group $\mathrm{SO}(4) \times \mathrm{SO}(4)'$ in the string theory is mapped to the product of the Lorentz (euclidean) symmetry and the R-symmetry, $\mathrm{SO}(4)_L \times \mathrm{SO}(4)_R$, in the field theory. Thus, on the field theory side, the $\mathbb{Z}_2$ symmetry interchanges the action of $\mathrm{SO}(4)_L$ with $\mathrm{SO}(4)_R$. A symmetry between spacetime and the internal (R-)space is quite novel. Since the $\mathbb{Z}_2$ symmetry is inherited from closed strings, it is natural to expect that the $\mathbb{Z}_2$ symmetry is respected by string interactions unlike that in [28]. If this is true, the supersymmetry of OD-branes descending from the closed string will be preserved even after introducing string interactions.

It should be interesting to generalize the classification of D-branes as done in this paper to other backgrounds, e.g., the Penrose limit of $AdS_5 \times S^5/\mathbb{Z}_N$, the type-IIA plane wave and the Gödel universe. We hope to address these problems in the near future.

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A. D-brane field equations

Here we will show, using the general D5-brane field equations derived in [14], that the modified Neumann boundary conditions in eqs. (5.35) and (5.38) can be realized by appropriately turning on a worldvolume flux.

In our case under consideration, $\Phi = B_{ij} = C_n^{RR} = 0$ except as C_4^{RR} giving rise to the background $F_5 = dC_4^{RR}$ in eq. (1.1).[8] The relevant worldvolume Chern-Simons coupling is then of the form

$$S_{WZ} = \tau_5 \int_{\Sigma_6} C_4^{RR} \wedge \widehat{F}_2 \tag{A.1}$$

[8] In this appendix we will use the notation adopted in [14] and set $2\pi\alpha' = 1$.

where $\widehat{F}_2$ is a gauge flux in the $D5$-brane worldvolume Σ_6. The equations of motion for a $D5$-brane read as (see eq. (3.2) in [14]):

$$\partial_i(\sqrt{-M}\theta^{ii_1}) = \frac{1}{5!}\epsilon^{i_1i_2i_3i_4i_5i_6}F_{i_2i_3i_4i_5i_6}\,, \tag{A.2}$$

$$\partial_i(\sqrt{-M}G^{ij}\partial_j X^n g_{mn}) = 0\,, \tag{A.3}$$

where $M_{ij} = g_{ij} + \widehat{F}_{ij}$ and

$$G^{ij} = \left(\frac{1}{g+\widehat{F}}g\frac{1}{g-\widehat{F}}\right)^{ij}, \tag{A.4}$$

$$\theta^{ij} = \left(\frac{1}{g+\widehat{F}}\widehat{F}\frac{1}{g-\widehat{F}}\right)^{ij}. \tag{A.5}$$

Let us first check the equations of motion (A.2) and (A.3) for the familiar example, $(+,-,4,0)$ $D5$-brane, which also serves to fix the relation between the worldvolume flux $\widehat{F}_2$ and the field strength F_2 entering in the worldsheet boundary coupling such as (5.42) and (5.44). The world volume coordinates are given by $\xi^i = (X^+, X^-, X^1, X^2, X^3, X^4)$. Since $\sqrt{-M} = 1$ and $G^{++} = 0$ in the metric (1.1), the left-hand side of eq. (A.3) identically vanishes and thus eq. (A.3) gives no constraint. However, eq. (A.2) gives some constraint for flat $(+,-,4,0)$ embedding since

$$\partial_r \widehat{F}^{-r} = 2\mu\,, \qquad (r = 1,\ldots,4) \tag{A.6}$$

where we used the fact that $\theta^{-r} = \widehat{F}^{-r}$. Now we identify

$$2\widehat{F}^{-r} = F^{-r} \tag{A.7}$$

from which we get[9]

$$F^{-r} = \mu X^r \tag{A.8}$$

which is exactly the Born-Infeld flux necessary to preserve the dynamical supersymmetry of $(+,-,4,0)$-brane [20].

Let us now check the equations of motion (A.2) and (A.3) for an $OD5$-brane, to be specific, described by $\Omega = \frac{1}{2}(\gamma^1-\gamma^6)(\gamma^2-\gamma^5)\gamma^{34}$ whose world volume coordinates $\xi^i = (X^+, X^-, X^{\hat{1}}, X^{\hat{2}}, X^{\dot{3}}, X^{\dot{4}})$ are given below eq. (3.1). Only eq. (A.2) gives a constraint for flat embedding ξ^i with flux:

$$\partial_r \widehat{F}^{-r} = \mu\,, \qquad (r = \hat{1},\hat{2} \text{ or } \dot{3},\dot{4})\,. \tag{A.9}$$

Note that the right-hand side of eq. (A.9) is reduced by half compared to the $(+,-,4,0)$-brane since the worldvolume of the $OD5$-brane, $\widehat{\Sigma}_6$, is slanted against the background RR 5-form $F_5 = dC_4^{RR}$. Using the relation (A.7), we get

$$F^{-\dot{r}} = \mu X^{\dot{r}}\,, \qquad \dot{r} = \dot{3},\dot{4}\,, \tag{A.10}$$

$$F^{-\hat{r}} = \mu X^{\hat{r}}\,, \qquad \hat{r} = \hat{1},\hat{2}\,, \tag{A.11}$$

[9]Indeed the factor 2 difference was originated from our different normalization between worldsheet light-cone coordinates defined by $\sigma^\pm = \frac{1}{2}(\tau\pm\sigma)$ and worldvolume light-cone coordinates given by $\xi^\pm = X^\pm = \frac{1}{\sqrt{2}}(X^0 \pm X^9)$.

which are the correct worldvolume fluxes necessary to preserve the dynamical supersymmetry of the $OD5$-brane as was shown in eqs. (5.42) and (5.44).

Similarly, we can apply the same method to an OD_+5-brane with flux. If the OD_+5-brane could preserve the dynamical supersymmetry, the Neumann boundary condition should be modified as $\partial_\sigma X^r - \mu X^r = 0$, $\forall r = (\hat{1}, \hat{2}, \hat{3}, \hat{4})$. However it is not possible since the equation (A.2) for the OD_+5-brane embedding with flux now requires

$$\partial_r F^{-r} = \mu \qquad \Leftrightarrow \qquad F^{-r} = \frac{\mu}{4} X^r \,. \tag{A.12}$$

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